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PENETRATION STUDIES OF ICE WITH APPLICATION TO ARCTIC AND SUBARCTIC WARFARE

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Perforation problems were treated with the aid of a two-dimensional, large deformation, dynamic, elastic-plastic computer code (CANDIA CODE) which was developed in a previous investigation. Specifically, axi-symmetric, dynamic stress distributions were studied under conditions of impact at normal incidence for a cylindrical blunt end projectile and a sea ice target slab. A capability for considering perforation problems where fracture in the sea ice target material occurs was developed. In this connection, the redistributions of both tensile and shear stresses that accompany propagating fracture surfaces were acknowledged. The program was also modified to provide projectile deceleration loads during the perforation process, and subroutines were created to treat multilayer ice slabs and thereby accommodate effects contributed by overlying snow cover and the underlying mushy sea ice skeleton layer.

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PREFACE

The investigation reported here was sponsored by the U.S. Naval Ordnance Laboratory (NOL, White Oak), Silver Spring, Maryland, under ONR Contracts Nonr-2332(00) and N00014-68-A-0243 (SRI Project No. 7000-452). The work was performed between 1 July 1969 - 31 December 1969 and 15 April 1970 - 31 June 1970 and is designated, unofficially, Phase IV Study. Results of the Phase I, Phase II, and Phase III studies were published in separate Stanford Research Institute reports under a similar title and dated November 1965, May 1967, and September 1969.

The contract was monitored for the Naval Ordnance Laboratory by Mr. M. M. Kleinerman. Project leader for Stanford Research Institute was Dr. Bernard Ross, and principal investigators were Dr. Sathya Hanagud, Dr. Bernard Ross, and Mr. Gursharan Sidhu.

CONTENTS

ILLUSTRATIONS

SYMBOLS

A - cross-section area of projectile

a - radius of spherical cavity, see Fig. 3

B_1, B_2 - constants related to dynamic pressure, see Eq. III-34

b - radius of spherical shock front, see Fig. 3

C_o - constant of integration

C_1, C_2 - constants related to dynamic pressure, see Eq. III-77

D - diameter of projectile

E - modulus of elasticity (Young's modulus)

E_t - tangent modulus for linear strain-hardening

$\bar{f}$ - limit of integration, see Eqs. III-8, III-41

$f(t)$, $g(t)$ - functions of integration

k - radius of elastic-plastic spherical shock front, see Fig. 5

M - mass of projectile

n - summation exponent

P_I - static pressure term given by Eq. III-52

P_{II} - static pressure term given by Eq. III-72

p_s - static pressure term given by Eq. III-34

$p(t)$ - dynamic pressure applied to spherical cavity surface as a function of time

$q, \dot{q}, \ddot{q}$ - depth, velocity, and acceleration, respectively, of projectile in target material

q_{o_s} - depth of projectile penetration at transition point, $q_{o_s} = R$

q_t - final penetration depth of projectile

R - depth of snow-ice interface at transition point of projectile penetration process, see Fig. 2

R_o - depth of snow cover, see Fig. 1

r - radial coordinate

r' - dummy variable of integration

t - time

V_o - velocity of projectile after completion of shallow penetration phase, assumed equal to impact velocity

v - outward particle velocity in radial direction

v_{o_s} - projectile velocity at transition point, equal to impact velocity for second phase of penetration

x - nondimensional quantity, $x = \xi^3$

Y - yield stress

y - nondimensional quantity, $y = 1/x$

α_p - ice parameter given by $\alpha_p = \left(1 - \frac{\rho_o}{\rho_{\ell_p}}\right)$

$\bar{\alpha}_p$ - parameter given by $\bar{\alpha}_p = \left(1 - e^{-3\beta}\frac{\rho_o}{\rho_{\ell_p}}\right)$

α_s - snow parameter given by $\alpha_s = \left(1 - \frac{\rho_{o_s}}{\rho_{\ell_s}}\right)$

$\bar{\alpha}_s$ - parameter given by $\bar{\alpha}_s = \left(1 - e^{-3\beta_s} \frac{\rho_{o_s}}{\rho_{\ell_p}}\right)$

α_{ℓ_s} - parameter given by $\alpha_{\ell_s} = \left(1 - \frac{\rho_{\ell_p}}{\rho_{\ell_s}}\right)$

β - material parameter for ice given by $\beta = \frac{Y}{2E} - \frac{\bar{\epsilon}_{\ell_p}}{3}$

β_s - material parameter for snow given by $\beta_s = \frac{Y_s}{2E_s} - \frac{\bar{\epsilon}_{\ell_s}}{3}$

ϵ - small quantity in the mathematical sense

ϵ_ℓ, $\bar{\epsilon}_\ell$ - locking strain, $\bar{\epsilon}_\ell = -\epsilon_\ell$

ϵ_r, ϵ_θ - normal strains in radial and circumferential directions, respectively

$\dot{\epsilon}_r$, $\dot{\epsilon}_\theta$ - normal strain rates in radial and circumferential directions, respectively

η - series expression given by $\eta = \sum_{n=1}^{\infty} \frac{1}{n^2} \left(1 - e^{-3\beta} \frac{\rho_o}{\rho_{\ell_p}}\right)^n$

θ - equatorial spherical coordinate

ξ - nondimensional quantity, $\xi = r/a$

ρ - density of target material

ρ_ℓ - locking density of target material

σ_r, σ_θ - normal stresses in radial and circumferential directions, respectively

Subscripts

p - refers to locked plastic region in sea ice

o - refers to initial values and/or stress-free region

s - refers to snow region

I - refers to ice region

1, 2 - initial and final states, respectively, of a hydrostatic compression process

I INTRODUCTION

This report presents the final results of a comprehensive, long term investigation concerning the penetration and perforation* behavior of Arctic sea ice subjected to impact by inert projectiles. The importance of these engineering problems to the Arctic Antisubmarine Warfare Program has been described in three reports under the present title issued during November 1965,[1]** May 1967,[2] and September 1969.[3] The last report contained a complete summary of work performed and progress achieved up to that date; therefore, the topic is not considered anew.

However, it is noted briefly that two important analytical procedures for treating projectile-sea ice interaction problems were developed after completion of the second study. As described in the last report, these efforts included the realization of a large deformation, elastic-plastic, artificial viscosity type computer program (CANDIA CODE) which could be used to solve a wide variety of axisymmetric, two-dimensional, dynamic, and/or impact problems,[4,5,6,7,8] and a large deformation deep penetration theory for analyzing projectile motion in compactible media.[9,10,11,12]

Even though it was possible to derive a substantial body of information by applying these procedures to Arctic sea ice perforation

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* For the purpose of this study, penetration can be defined as the entrance of a projectile into a target or ice cover without completing its passage through the body, whereas perforation implies the complete piercing of the target slab or ice cover by the projectile.

** Superscript numbers refer to references which are collected at the end of this report.

and penetration problems, it was recognized that further improvements were still required for practical application of the theoretical work.

For example, perforation of an Arctic sea ice cover by an impacting projectile creates fracture surfaces in the target material, and it is the resultant geometry of this process that describes both the mode of perforation and the associated value of critical impact velocity.* Moreover, important redistributions of stresses occur when fracture surfaces develop, and these changes affect both the failure phenomenon itself and the magnitudes of stress experienced by the impacting projectile. In this connection, the previous version of the code was capable of providing complete descriptions of both stress and deformation fields in the impacting bodies. It remained to postulate suitable failure criteria and expected crack propagation paths to define the global fracture pattern and the mechanism of projectile transit through the sea ice cover.

In consequence of the present work, it is possible now to track the actual failure configuration as a function of time after initial impact, and to acknowledge that the complement of both shear and cleavage fracture surface propagation paths is being influenced continuously by the developing fracture process.

On the other hand, the previous deep penetration theory was limited in application to homogeneous, isotropic media and thus was not suitable for cases involving a snow covered Arctic ice sheet. To correct this shortcoming, present research efforts were directed toward realization of a multilayer deep penetration theory to treat the practical projectile

* Critical velocity is defined as the minimum or threshold impact velocity for complete penetration, i.e., perforation.

penetration problem characterized by a two-layer, snow-ice target material. This work can be employed to obtain penetration performance curves for projectiles deployed in a typical Arctic environment.

In sum, these connected analyses (namely, direct treatment of the dynamic fracture problem and synthesis of a multilayer, large deformation, deep penetration theory) constitute the basis of this investigation. However, in addition to these studies a subsidiary effort concerning the determination of maximum deceleration loads experienced by a projectile during the sea ice perforation process was carried out. Unfortunately, numerical calculations for both of these problems could not be carried out due to a lack of funds for computer runs. Therefore, the significance of present developments as applied to the design and development of naval weapons for Arctic application could not be examined.

II MECHANICS OF FRACTURE

The work accomplished was concentrated on modifying and restructuring the CANDIA CODE to enable theoretical analysis of problems where fracture of the ice slab occurs. This phenomenon engenders redistributions of stresses in the target material, and accompanying effects in the magnitude and distributions of loads sustained by the impacting projectile. Thus, from an overall viewpoint, these efforts were directed toward the transformation of a computer code employed previously as a research tool to a working code which can be utilized to perform engineering design and development calculations.

To sustain these particular goals and to treat the fracture problem in sea ice, per se, it was necessary to recast the code so that problems characterized by greater numbers of variables and finer finite-difference calculation mesh networks could be accommodated. For example, in initial format the code considered sea ice to be homogeneous and isotropic, whereas, in fact, this material is strongly anisotropic and inhomogeneous. Consequently, introduction of this more advanced description of material composition would furnish a demand for increase in the number of input variables that are needed for numerical calculation. Moreover, greater machine storage and capability were also required because a developing fracture surface submits many more mesh points of importance into the calculation process. To overcome these difficulties, the entire CANDIA CODE, as listed and flow-charted in the last comprehensive report,[3] was restructured to eliminate its dependence on high speed core memory storage. As a result, the code in its present format can be employed to take advantage of random access disc storage.

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At this time, the entire code exists on disc in precompiled form which is compatible with all electronic data processing equipment based on the FORTRAN language. As such, the code gains advantage in capability and versatility by employing fixed or consistent amounts of high speed core storage with a random access interface.

Briefly, advantages of the new disc format are:

- Capacity increased; that is, problems characterized by greater numbers of intrinsic variables and requiring finer mesh works for calculation purposes can be treated. The fracture problem in sea ice falls within this definition.
- Convenience increased; that is, ease of working with the code has been considered. For instance, the code is transcribed presently on one disc pack in lieu of multiple racks of perforated cards.
- Versatility increased; that is, interrupted calculations can be performed so that when a computation procedure is stopped intermittently it is no longer necessary to resume calculations from the initial, or zero, input state. Instead, numerical treatment can be restarted from the previous termination point. In this manner, and with particular relevance to the fracture problem, an analysis can be intercepted to focus attention on the development of a propagating fracture surface and the attendant stresses in plastic zones surrounding a crack tip.
- Information display possibilities increased; that is, direct computer readout is feasible to furnish continuing computer code output by means of cathode ray terminals. Thus, it becomes practical to take movies of progressive display patterns and thereby indicate visually the continuing propagation of fracture surfaces and ancillary stress redistributions in both target material and impacting projectile.

With restructuring and modification of the computer code effected, it was possible to attack the fracture problem directly. However, in this case it was acknowledged first that the treatment of stress redistributions due to developing fracture surfaces would be closely coupled to the states of stress existing on free surface boundaries such that the proper numerical treatment of boundary conditions themselves would be an utmost necessity. In this connection, it was pointed out in Ref. 3 that many large deformation elastic-plastic computer codes currently in existence were founded on incorrect boundary condition formulations. Thus, an appendix was included in the report containing a lengthy derivation for the true treatment of these boundary conditions. Because this development was somewhat unwieldy in the form presented, considerable effort was expended during the current period in rewriting a more concise and simplified algorithm for this aspect of the problem.

Once this task was completed, it was possible to create an additional algorithm to treat the fracture problem itself. In this case, the algorithm was founded on the development of a point condition code which enabled integer point condition values to be associated with different characteristic free surface points on nodal intersections in the finite difference network. For example, individual point values were ascribed with concomitant subroutines to represent lateral and transverse free surface points, interior fracture surface points, and corner points on both target and projectile. As a result, point condition changes could be related to responsive subroutines in correspondence with physical transformations of both impacting bodies. Consequently, it is possible now to track in the computer both current boundary states and conditions, as well as emerging and propagating fracture surfaces.

Use was then made of the modified boundary condition and fracture algorithms to treat the shear fracture problem encountered when a blunt or hemispherical nose projectile experiences initial penetration in a target material. In this case, it was known from previous experimental, field test, and theoretical studies that, due to the anisotropic nature of a sea ice target material, initial fracture under these impact conditions would occur in the shear mode along σ_{zz}-normal stress planes in the vertical direction.*

With the shear fracture phase of the overall perforation process accounted for, work progressed toward realization of a fracture algorithm to describe failure surface propagation under tensile principal stresses. The important difficulty overcome in treating this problem was that fracture does not occur necessarily along a mesh direction.** Therefore, there remained the burden of outlining and defining not only the stress redistributions at incipient fracture but the directions of the fracture planes themselves.

In this context, a method of approach was devised and implemented in the present version of the code. Specifically, all components of the σ_{ij}

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* Fracture configurations in the shear mode are initiated upon projectile impact at normal incidence because, for a blunt penetrator, regions of intense shear stress are generated around the projectile periphery. Moreover, laboratory experiments and field test results, as well as theoretical calculations (CANDIA CODE), indicate that continued shear penetration and concomitant fracture patterns are engendered in the sea ice slab until penetration depths of the order of 70-80% sea ice sheet thickness are achieved. At this point, it is recognized that shear stress magnitudes fall off and that further fracture and ultimate perforation are the result of sea ice failure under tensile stress. In sum, complete penetration is obtained through a composite process characterized by initial failure due to shear and terminal failure as a result of tensile or cleavage fracture. Consequently, ultimate perforation results in the ejection of a cylindrical-conical shear plug of target material by the impacting body.

** In this connection, the orientations of component segments of a developing tensile fracture surface are related directly to the magnitudes and directions of local principal stresses, which themselves are dependent variables of the problem in question. Thus, developing cleavage fracture surfaces do not correspond necessarily in spatial alignment and configuration to the postulated geometry of the mesh network employed for finite-difference calculations.

stress tensor are determined at the center point of a given area segment in a mesh network. Knowing these quantities enables the three principal stresses to be determined, whereupon a test is performed to isolate any possibility that the shear stress component of the total stress tensor violates the allowable shear failure stress of the target material. Assuming this is not the case and that shear fracture does not develop, then the calculation sequence is pursued further by comparing the maximum principal stress value with the allowable failure stress in uniaxial tension. If the former quantity exceeds the latter, it can be assumed (i.e., at least for an elastic brittle material such as freshwater ice) that fracture occurs over the mesh section of interest. Then, spatial orientation of the fracture surface segment is obtained by considering the cleavage plane to be orthogonal to the direction of the maximum principal stress component.

Thus, knowledge of the magnitudes and directions of principal stress quantities at a central point in the mesh section enables the prediction of fracture likelihood and resultant local failure surface geometry to be made. Once fracture occurs over a mesh section, it is necessary to postulate free surface boundary conditions which provide a zero traction state over the developing area of material separation. Then, the physical situation at the failed mesh section must be rechecked at each and every time step of succeeding numerical calculation to determine whether changes in external loading and/or internal stress state have resulted in subjecting the mutual fracture surfaces to closure and compressive pressure. If this turns out to be the case, free surface boundary conditions are stricken and appropriate interface conditions substituted in their place. Finally, at the conclusion of each calculation time step, a locus or contour is passed through all of the individual fracture surface segments to indicate instantaneous states of the composite failure pattern developed in the target material subjected to projectile impact.

III MECHANICS OF PENETRATION

In addition to continuing research in the fracture problem, efforts were devoted to realizing a theory for the deep penetration of projectiles in multilayered media such as a snow covered Arctic sea ice sheet.[13,14,15] Solutions of the dee penetration problem for a single layer, homogeneous, isotropic sea ice cover have been obtained already, and their formulation and development were presented in a recently issued comprehensive report.[3] Fortunately, most of this previous work was applicable directly to the multilayered penetration problem; thus, the compounding of a tractable theory applicable to projectile-sea ice interaction problems for a more complicated target material did not offer substantial difficulty.

In this case, continued use was made of the two most important conceptual fundamentals upon which the previous theory was based. These are: first, that transformation of the deep penetration problem itself can be effected successfully to yield a problem concerned with dynamic stress distributions in an infinite solid that contains a spherical cavity subjected to step input in pressure*, and, second, that the dynamic constitutive behavior of compactible materials such as Arctic sea ice and snow can be represented satisfactorily by assuming an idealized locking approximation for Rankine-Hugoniot material behavior under hydrostatic compressive stress.

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* This metamorphosis is founded specifically on the assumption that dynamic pressure experienced by a penetrating projectile at its stagnation point is comparable by reason of symmetry to the similar pressure quantity which exists at the surface of a dynamic expanding spherical cavity. A corollary assumption enables an expression for the complete force resisting continued penetration to be established by postulating that the spatial variation of pressure over the frontal portion of the projectile can be represented by a simple cosine relationship.[17]

Both of these arguments were invoked to resolve the multilayer penetration problem. In particular, a systematic approach was devised to recast the two-layer target configuration into the form of an idealized infinite body that included a spherical cavity surrounded by two concentric spherical regions of material having the relevant physical and mechanical properties of Arctic snow and sea ice. Then, the dynamic spherical expansion problem was worked through again for the composite material configuration. This task demanded relatively more complicated algebraic formulations but did not require an increase in mathematical generality or important theoretical innovation. The problem was made somewhat easier by the fact that reflected stress waves in a locked material travel at infinite speeds, so that introduction of limiting conditions into the mathematical process resulted in well-defined simplifications.

The problem considered is depicted schematically in Figs. 1 and 2. Attention is directed first to the ideal locked-rigid plastic material in Region 1_s (Figs. 3 and 4) which is behind the advancing shock front at $r = b(t)$. The locking condition on strain is

$$\epsilon_r + 2\epsilon_\theta = \epsilon_{\ell_s} \tag{III-1}$$

where ϵ_{ℓ_s} the ideal locking strain for snow is a time-invariant material property. Differentiating with respect to time results in

$$\dot{\epsilon}_r + 2\dot{\epsilon}_\xi = 0 \tag{III-2}$$

These strain rate quantities can be expressed in terms of radial outward particle velocity, $v(r,t)$, by the kinematic relations

$$\dot{\epsilon}_r = \frac{\partial v}{\partial r} \qquad \dot{\epsilon}_\theta = \frac{v}{r} \tag{III-3}$$

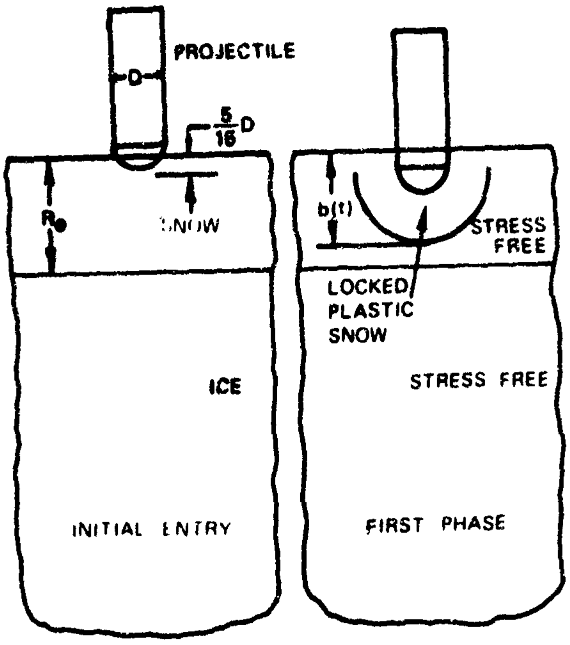

FIGURE 1 SCHEMATIC REPRESENTATION OF PROJECTILE PENETRATION PROCESS IN SNOW COVERED ICE SHEET. INITIAL ENTRY AND FIRST PHASE MOTION

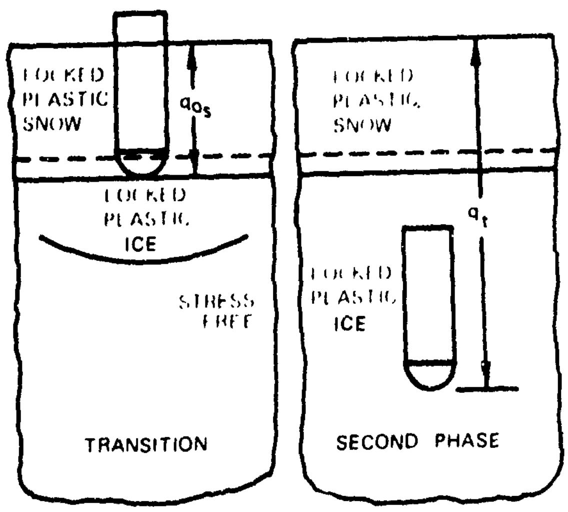

FIGURE 2 SCHEMATIC REPRESENTATION OF PROJECTILE PENETRATION PROCESS IN SNOW COVERED ICE SHEET TRANSITION AND SECOND PHASE MOTION

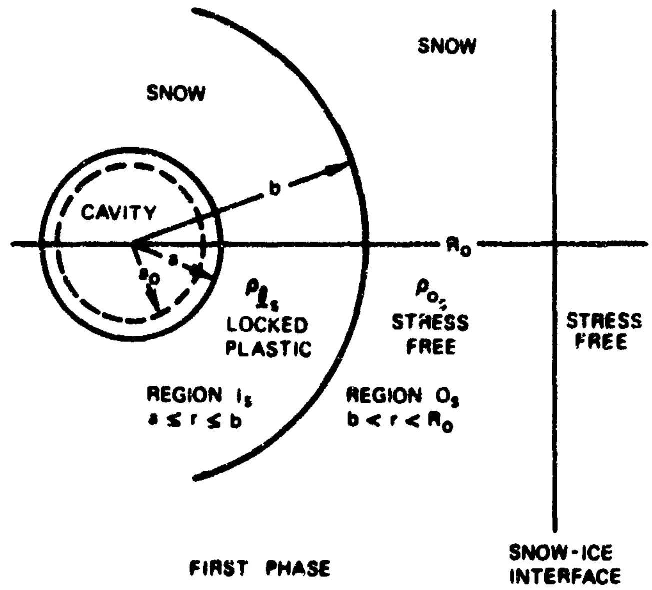

FIGURE 3 DYNAMIC CAVITY EXPANSION PROBLEM FOR A RIGID PLASTIC, IDEAL LOCKING MATERIAL

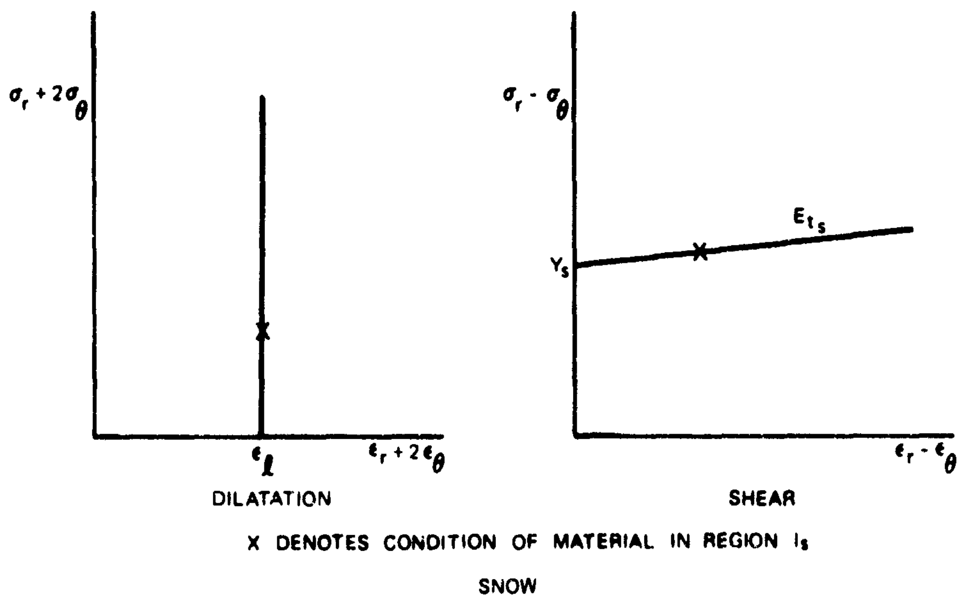

FIGURE 4 IDEALIZED STRESS-STRAIN CURVES FOR A RIGID PLASTIC, IDEAL LOCKING MATERIAL WITH LINEAR STRAIN HARDENING AFTER YIELD

Substitution and integration yield

$$v_s = \frac{f_s(t)}{r^2} \qquad \text{(III-4)}$$

where subscript s denotes quantities in the locked-plastic snow region. The stress-strain relationship in shear for a rigid-plastic material with linear strain hardening is[3]

$$\sigma_\theta - \sigma_r = Y_s + \frac{2}{3} E_{t_s}(\epsilon_\theta - \epsilon_r) \qquad \text{(III-5)}$$

Under conditions of spherical symmetry, the equation of motion becomes

$$\frac{\partial \sigma_r}{\partial r} + \frac{2}{r}(\sigma_r - \sigma_\theta) = \rho_{\ell_s}\left(\frac{\partial v_s}{\partial t} + v_s \frac{\partial v_s}{\partial r}\right) \qquad \text{(III-6)}$$

Finally, the large deformation circumferential strain is given by

$$\epsilon_\theta = \ln \frac{r}{r_o} \qquad \text{(III-7)}$$

Combining these relationships into Eq. III-6 and integrating provides the following expression.

$$\sigma_{r_s} = 2Y_s \ln r + 2\int_{\bar{f}_s}^{r} \frac{1}{r'} E_{t_s}\left(2 \ln \frac{r'}{r_o} + \frac{2\bar{\epsilon}_{\ell_s}}{3}\right)dr'$$

$$- \rho_{\ell_s} \frac{\dot{f}_s}{r} + \frac{1}{2}\rho_{\ell_s} \frac{f_s^2}{r^4} + g_s(t) \qquad \text{(III-8)}$$

Conservation of mass across the shock front at the stress free-locked plastic interface yields

$$\rho_{o_s}\dot{b} = \rho_{\ell_s}(\dot{b} - v_s) \quad @ \quad r = b(t) \qquad \text{(III-9)}$$

or upon reduction

$$f_s(t) = \alpha_s b^2 \dot{b} \tag{III-10}$$

where

$$\alpha_s = 1 - \rho_{o_s}/\rho_{\ell_s} \tag{III-11}$$

Similarly, conservation of momentum yields

$$\sigma_{r_s} = -\rho_{\ell_s}(\dot{b} - v_s)v_s \quad @ \quad r = b(t) \tag{III-12}$$

or upon reduction

$$\sigma_{r_s}\Big|_{r=b} = -\rho_{o_s}\alpha_s \dot{b}^2 \tag{III-13}$$

The boundary condition on stress at the cavity surface is

$$\sigma_{r_s} = -p(t) \quad @ \quad r = a(t) \tag{III-14}$$

Then, from Eq. III-8 evaluated at $r = a(t)$

$$g_s(t) = -p(t) - 2Y_s \ln a - 2\int_{\bar{f}_s}^{a} \frac{1}{r'} E_{t_s}\left(2 \ln \frac{r'}{r_o} + \frac{2\bar{\epsilon}_{\ell_s}}{3}\right)dr'$$

$$+ \rho_{\ell_s}\frac{\dot{f}_s}{a} - \frac{1}{2}\rho_{\ell_s}\frac{f_s^2}{a^4} \tag{III-15}$$

so that the equation for radial stress becomes

$$\sigma_{r_s} = - p(t) + 2Y_s \ln \frac{r}{a} + 2 \int_a^r \frac{1}{r'} E_{t_s} \left(2 \ln \frac{r'}{r_o} + \frac{2\bar{\epsilon}_{\ell_s}}{3}\right) dr'$$

$$+ \rho_{\ell_s} \alpha_s \left(b^2 \ddot{b} + 2b\dot{b}^2\right) \left(\frac{1}{a} - \frac{1}{r}\right)$$

$$- \frac{1}{2} \rho_{\ell_s} \alpha_s^2 b^4 \dot{b}^2 \left(\frac{1}{a^4} - \frac{1}{r^4}\right) \qquad \text{(III-16)}$$

Using Eq. III-13, an expression is obtained for the pressure variation with time at the cavity surface

$$p(t) = 2Y_s \ln \frac{b}{a} + 2 \int_a^b \frac{1}{r'} E_{t_s} \left(2 \ln \frac{r'}{r_o} + \frac{2\bar{\epsilon}_{\ell_s}}{3}\right) dr'$$

$$+ \alpha_s \rho_{\ell_s} \left(b^2 \ddot{b} + 2b\dot{b}^2\right) \left(\frac{1}{a} - \frac{1}{b}\right)$$

$$- \frac{1}{2} \alpha_s^2 \rho_{\ell_s} b^4 \dot{b}^2 \left(\frac{1}{a^4} - \frac{1}{b^4}\right) + \rho_{o_s} \alpha_s \dot{b}^2 \qquad \text{(III-17)}$$

Equation III-17 can be reduced further by application of the boundary condition on particle velocity at the cavity surface

$$v_s = \dot{a} \quad @ \quad r = a(t) \qquad \text{(III-18)}$$

Using Eqs. III-4, III-10 results in

$$\alpha_s b^2 \dot{b} = a^2 \dot{a} \qquad \text{(III-19)}$$

Differentiation of both sides of Eq. III-19 with respect to time yields

$$\alpha_s\left(b^2\ddot{b} + 2b\dot{b}^2\right) = a^2\ddot{a} + 2a\dot{a}^2 \qquad \text{(III-20)}$$

Substitution of these results into Eq. III-17 gives

$$p(t) = 2Y_s \ln\frac{b}{a} + 2\int_a^b \frac{1}{r'} E_{t_s}\left(2\ln\frac{r'}{r_o} + \frac{2\bar{\epsilon}_{\ell_s}}{3}\right)dr'$$

$$+ \rho_{\ell_s}\left(a^2\ddot{a} + 2a\dot{a}^2\right)\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$- \frac{1}{2}\rho_{\ell_s} a^4\dot{a}^2\left(\frac{1}{a^4} - \frac{1}{b^4}\right) + \rho_{o_s}\frac{a^2}{b^2}\dot{a}\dot{b} \qquad \text{(III-21)}$$

Surfaces of discontinuity in displacement and particle velocity cannot exist if fracture at the shock front is omitted from consideration. Thus, $v_s(t)$ is zero on the shock front since the stress free material ahead of the locked plastic zone is quiescent. As a consequence, no material moves through the shock front and the following compressibility relationship can be applied at successive times over the volume of locked plastic material bounded by the cavity surface and an arbitrary spherical surface at the present shock front location

$$(b^3 - a^3)\rho_{\ell_s} = (b^3 - a_o^3)\rho_{o_s} \qquad \text{(III-22)}$$

Upon reduction

$$\alpha_s\frac{b^3}{a^3} = 1 - \frac{a_o^3}{a^3}\frac{\rho_{o_s}}{\rho_{\ell_s}} \qquad \text{(III-23)}$$

But, it has been shown by Goodier[16,17,18] that, for the deep penetration problem, the following assumption can be made

$$a^3 \gg a_o^3$$

In addition, $\rho_\ell > \rho_o$, so that

$$b^3 \cong \frac{1}{\alpha_s} a^3 \tag{III-24}$$

Equation III-21 becomes

$$p(t) = -\frac{2}{3} Y_s \ln \alpha_s + 2 \int_a^b \frac{1}{r'} E_{t_s} \left(2 \ln \frac{r'}{r_o} + \frac{2\bar{\epsilon}_{\ell_s}}{3}\right) dr'$$

$$+ \rho_{\ell_s} \left(a\ddot{a} + 2\dot{a}^2\right)\left(1 - \alpha_s^{1/3}\right)$$

$$- \frac{1}{2} \rho_{\ell_s} \dot{a}^2 \left(1 - \alpha_s^{4/3}\right) + \rho_{o_s} \dot{a}^2 \alpha_s^{1/3} \tag{III-25}$$

The remaining integral must be evaluated. Application of an appropriate compressibility relationship at an arbitrary location, r, within the locked plastic region yields

$$(r^3 - a^3)\rho_{\ell_s} = (r_o^3 - a_o^3)\rho_{o_s} \tag{III-26}$$

Using the deep penetration assumption (i.e., $a^3 >> a_o^3$), Eq. III-26 becomes*

$$\frac{r}{r_o} = \left[\frac{\rho_{o_s}}{\rho_{\ell_s}} \; \frac{r^3/a^3}{r^3/a^3 - 1}\right]^{1/3} \qquad \text{(III-27)}$$

Letting

$$x = \xi^3 = r^3/a^3$$

and employing Eq. III-27 enables the integral expression in Eq. III-25 to be written as follows

$$2\int_{a(t)}^{b(t)} \frac{1}{r'} E_{t_s}\left(2 \ln \frac{r'}{r_o} + \frac{2\bar{\epsilon}_{\ell_s}}{3}\right)dr'$$

$$= \frac{4}{9} E_{t_s}\left\{\int_1^{b^3/a^3} \ln \frac{\rho_{o_s}}{\rho_{\ell_s}} \frac{dx}{x} + \int_1^{b^3/a^3} \bar{\epsilon}_{\ell_s} \frac{dx}{x} + \int_1^{b^3/a^3} \ln \frac{x}{x-1} \frac{dx}{x}\right\} \qquad \text{(III-28)}$$

---

*

$$\left(r^3 - a^3\right) \rho_{\ell_s} = \left(r_o^3 - a_o^3\right) \rho_{o_s}$$

$$r^3 - a^3 = \left(r_o^3 - a_o^3\right) \rho_{o_s}/\rho_{\ell_s}$$

$$r^3 - a^3 = r_o^3 \frac{\rho_{o_s}}{\rho_{\ell_s}} - a_o^3 \frac{\rho_{o_s}}{\rho_{\ell_s}}$$

$$\frac{r^3}{r_o^3} - \frac{a^3}{r_o^3} = \frac{\rho_{o_s}}{\rho_{\ell_s}} - \frac{a_o^3}{r_o^3} \frac{\rho_{o_s}}{\rho_{\ell_s}}$$

(footnote continued on next page)

Evaluation of the first two integrals in the second member of Eq. III-28 is straightforward.[3] The remaining integral is attacked by letting $y = 1/x$ and changing the lower limit of integration to $1-\epsilon$ where $\epsilon \ll 1$. There results

$$\int_1^{b^3/a^3} \ln \frac{x}{x-1} \frac{dx}{x} \rightarrow \int_{1-\epsilon}^{a^3/b^3} \ln (1-y) \frac{dy}{y} \qquad \text{(III-29)}$$

Since $b > a$ always and $1-\epsilon < 1$, the range of values over which integration takes place (i.e., $1-\epsilon \geq y \geq b^3/a^3$) ensures that $y < 1$ so that the integrand can be expanded in series form. Step by step integration yields

$$\int_1^{b^3/a^3} \ln \frac{x}{x-1} \frac{dx}{x} = \left[y + \frac{y^2}{4} + \frac{y^3}{9} + \ldots \right]_{a^3/b^3}^{1-\epsilon} \qquad \text{(III-30)}$$

---

* (footnote continued)

but $\rho_{\ell_s} > \rho_{o_s}$ and $r_o^3 \gg a_o^3$

$$\therefore \qquad \frac{\rho_{o_s}/\rho_{\ell_s}}{r^3/r_o^3 - a^3/r_o^3} = 1$$

$$\frac{(\rho_{o_s}/\rho_{\ell_s})\, r^3}{r^3/r_o^3 - a^3/r_o^3} = r^3$$

$$\frac{(\rho_{o_s}/\rho_{\ell_s})\, r^3\, r_o^3}{r^3 - a^3} = r^3$$

$$\therefore \qquad \frac{r^3}{r_o^3} = \frac{(\rho_{o_s}/\rho_{\ell_s})\, r^3/a^3}{r^3/a^3 - 1}$$

Allowing $\epsilon \to 0$ and introducing Eq. III-24 results in

$$\int_1^{b^3/a^3} \ln \frac{x}{x-1} \frac{dx}{x} = \frac{\pi^2}{6} - \sum_{n=1}^{\infty} \frac{1}{n^2} \alpha_s^n \qquad \text{(III-31)}$$

Finally, Eq. III-25 becomes

$$\begin{aligned} p(t) &= -\frac{2}{3} Y_s \ln \alpha_s + \frac{4}{9} E_{t_s} \left\{ \frac{\pi^2}{6} - \sum_{n=1}^{\infty} \frac{1}{n^2} \alpha_s^n \right\} \\ &\quad + \rho_{\ell_s} \left(a\ddot{a} + 2\dot{a}^2 \right) \left(1 - \alpha_s^{1/3} \right) \\ &\quad - \frac{1}{2} \rho_{\ell_s} \dot{a}^2 \left(1 - \alpha_s^{4/3} \right) + \rho_{o_s} \dot{a}^2 \alpha_s^{1/3} \end{aligned} \qquad \text{(III-32)}$$

The expression for pressure can be broken down into static and dynamic parts so that

$$p(t) = p_S + \rho_{\ell_s} \left(B_1 a\ddot{a} + B_2 \dot{a}^2 \right) \qquad \text{(III-33)}$$

where

$$p_S = \frac{2}{27} \pi^2 E_{t_s} - \frac{2}{3} Y_s \ln \alpha_s - \frac{4}{9} E_{t_s} \sum_{n=1}^{\infty} \frac{1}{n^2} \alpha_s^n$$

$$B_1 = 1 - \alpha_s^{1/3}$$

$$B_2 = \frac{3}{2} - (1 + \alpha_s) \alpha_s^{1/3} + \frac{1}{2} \alpha_s^{4/3}$$

or

$$B_2 = \frac{3}{2} - \alpha_s^{1/3} - \frac{1}{2} \alpha_s^{4/3} \qquad \text{(III-34)}$$

The relationship for pressure variation at the cavity surface given by Eq. III-33 is used in the Goodier deep penetration theory[3,16] to obtain an expression for resisting forces acting on the projectile. Then, an equation of motion for projectile transit in the target material can be written

$$M\ddot{q} = -\left\{p_S + \frac{2}{3}\rho_{\ell_s}\left(B_1 \frac{D}{2}\ddot{q} + B_2\dot{q}^2\right)\right\}\frac{\pi D^2}{4} \qquad \text{(III-35)}$$

After integration, Eq. III-35 becomes

$$\ln\left(p_S + \frac{2}{3}B_2\rho_{\ell_s}\dot{q}^2\right) = -\frac{4}{3}B_2\rho_{\ell_s}\frac{q}{\frac{M}{A} + B_1\frac{D}{3}\rho_{\ell_s}} + C_o \qquad \text{(III-36)}$$

Exact shallow penetration theory[3,16] is avoided in this consideration. Instead, it is assumed on balance that the onset of resistance to penetration in snow takes place at an indentation depth corresponding to the centroid of the nose hemisphere. That is, when $q = 5D/16$. Under these conditions, Eq. III-36 becomes

$$\ln\frac{p_S + \frac{2}{3}B_2\rho_{\ell_s}\dot{q}^2}{p_S + \frac{2}{3}B_2\rho_{\ell_s}V_o^2} = -\frac{4}{3}\frac{B_2\rho_{\ell_s}}{\frac{M}{A} + B_1\frac{D}{3}\rho_{\ell_s}}(q - 5D/16) \qquad \text{(III-37)}$$

The first phase of penetration is completed when the projectile encounters the snow-ice interface. Here,

$$q = q_{o_s} = R \gtrsim R_o$$

Substitution of $q = R$ into Eq. III-37 determines the projectile velocity, $\dot{q}_{o_s}$, which characterizes initiation of the second penetration phase.

In the second phase of penetration, the projectile encounters an ice cover which is assumed in the present problem to behave as an elastic-plastic solid in shear, and incompressible under hydrostatic stress. Attention is directed to Figs. 5 and 6. Using expressions which have been developed in Ref. 3, the radial normal stress at the snow-ice interface is given by*

$$
\begin{aligned}
-\sigma_{r_I}\Big|_{r=R} = & -\frac{2}{3} Y \ln\left(1 - e^{-3\beta}\frac{\rho_o}{\rho_{\ell_p}}\right) + \frac{2}{27}\pi^2 E_t \\
& - \frac{4}{9} E_{t_s} \sum_{n=1}^{\infty} \frac{1}{n^2}\left(1 - e^{-3\beta}\frac{\rho_o}{\rho_{\ell_p}}\right)^n \\
& + \frac{4}{9} E\left(1 - e^{-3\beta}\right) + \alpha_p \rho_o \dot{b}^2 \\
& + \alpha_p \rho_{\ell_p}\left(b\ddot{b} + 2\dot{b}^2\right)\left(\frac{b}{R} - 1\right) \\
& - \frac{1}{2}\alpha_p^2 \rho_{\ell_p} \dot{b}^2 \left(\frac{b^4}{R^4} - 1\right)
\end{aligned}
\qquad \text{(III-38)}
$$

Again, from previous work

$$
v_I\Big|_{r=R} = \alpha_p \frac{b^2\dot{b}}{R^2} = \frac{f_I(t)}{R^2} \qquad \text{(III-39)}
$$

where $$f_I(t) = \alpha_p b^2 \dot{b} \qquad \text{(III-40)}$$

---

* The subscript, I, has been omitted from the quantities Y, ρ_o, ρ_{ℓ_p}, E_t, and E for reasons of mathematical convenience.

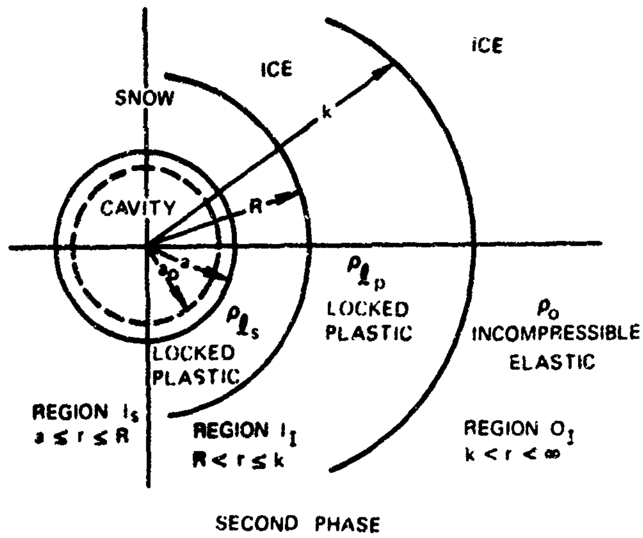

FIGURE 5 DYNAMIC CAVITY EXPANSION PROBLEM FOR AN ELASTIC-PLASTIC, INCOMPRESSIBLE MATERIAL

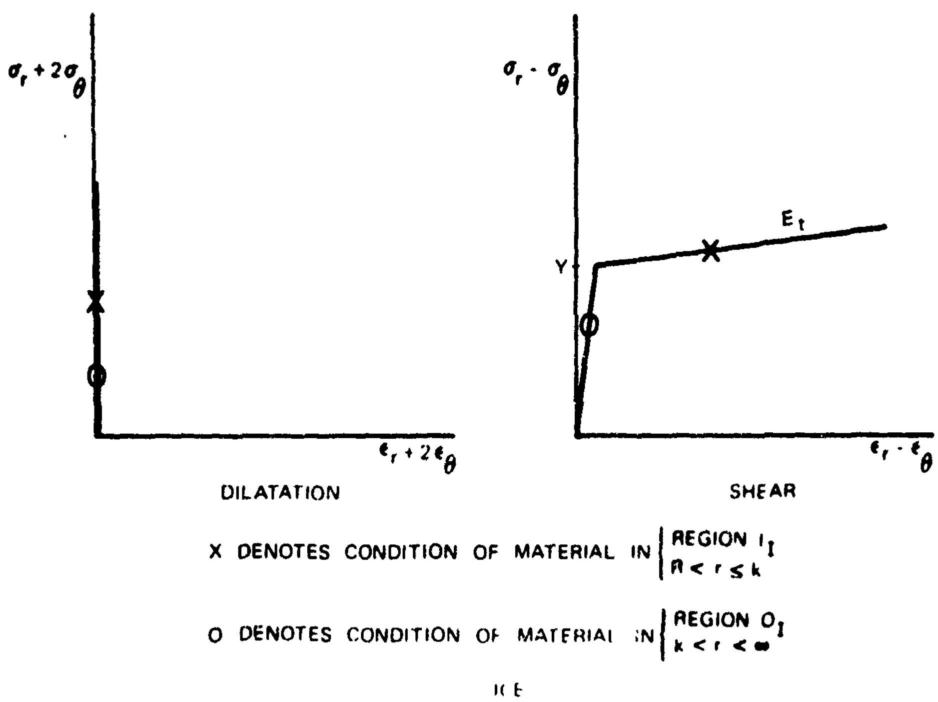

FIGURE 6 IDEALIZED STRESS-STRAIN CURVES FOR AN ELASTIC-PLASTIC, INCOMPRESSIBLE MATERIAL WITH LINEAR STRAIN HARDENING AFTER YIELD

In the snow region, the general solution to the problem can be written as

$$\sigma_{r_s} = 2Y_s \ln r + 2 \int_{\bar{f}_s}^{r} \frac{1}{r'} E_{t_s} \left(2 \ln \frac{r'}{r_o} + \frac{2}{3} \bar{\varepsilon}_{\ell_s}\right) dr'$$

$$- \rho_{\ell_s} \frac{\dot{f}_s}{r} + \frac{1}{2} \rho_{\ell_s} \frac{f_s^2}{r^4} + g_s(t) \qquad \text{(III-41)}$$

$$v_s = \frac{f_s(t)}{r^2} \qquad \text{(III-42)}$$

Boundary conditions are given by:

$r = a(t)$

$$\sigma_{r_s} = - p(t) \qquad \text{(III-43)}$$

$r = R(t)$

$$\rho_{\ell_s}(\dot{R} - v_s) = \rho_{\ell_p}(\dot{R} - v_I) \qquad \text{(III-44)}$$

$$\sigma_{r_s} - \sigma_{r_I} = - \rho_{\ell_p}(v_I - \dot{R})(v_I - v_s) \qquad \text{(III-45)}$$

The quantities σ_{r_I} and v_I at $r = R$ are given by Eqs. III-38, III-39. Use of Eq. III-43 enables the quantity, $g_s(t)$, in Eq. III-41 to be expressed in terms of $p(t)$.

Then, the new relationship for σ_{r_s} is given by

$$\sigma_{r_s} = -p(t) + 2Y_s \ln\frac{r}{a} + 2\int_a^r \frac{1}{r'} E_{t_s}\left(2\ln\frac{r'}{r_o} + \frac{2}{3}\bar{\epsilon}_{\ell_s}\right)dr'$$

$$+ \rho_{\ell_s}\dot{f}_s\left(\frac{1}{a} - \frac{1}{r}\right) - \frac{1}{2}\rho_{\ell_s} f_s^2\left(\frac{1}{a^4} - \frac{1}{r^4}\right) \tag{III-46}$$

From the boundary condition of Eq. III-44,

$$v_s\Big|_{r=R} = \left(1 - \frac{\rho_{\ell_p}}{\rho_{\ell_s}}\right)\dot{R} + \frac{\rho_{\ell_p}}{\rho_{\ell_s}} v_I \tag{III-47}$$

Then,

$$f_s(t) = \left(1 - \frac{\rho_{\ell_p}}{\rho_{\ell_s}}\right)R^2\dot{R} + \frac{\rho_{\ell_p}}{\rho_{\ell_s}} f_I(t) \tag{III-48}$$

Substitution into the boundary condition given by Eq. III-45 yields

$$\sigma_{r_s} - \sigma_{r_I}\Big|_{r=R} = \rho_{\ell_p}\left(\dot{R} - \alpha_p\frac{b^2\dot{b}}{R^2}\right)\left(\alpha_p\frac{b^2\dot{b}}{R^2} - \alpha_{\ell_s}\dot{R} - (1-\alpha_{\ell_s})\alpha_p\frac{b^2\dot{b}}{R^2}\right) \tag{III-49}$$

or

$$\sigma_{r_s} - \sigma_{r_I}\Big|_{r=R} = \rho_{\ell_p}\left(-\alpha_{\ell_s}\dot{R}^2 + 2\alpha_{\ell_s}\alpha_p\frac{b^2\dot{b}\dot{R}}{R^2} - \alpha_p^2\alpha_{\ell_s}\frac{b^4\dot{b}^2}{R^4}\right) \tag{III-50}$$

Substitution of Eqs. III-46, III-38 in Eq. III-50 gives

$$- p(t) + 2Y_s \ln \frac{R}{a} + 2 \int_a^R \frac{1}{r'} E_{t_s} \left(2 \ln \frac{r'}{r_o} + \frac{2}{3} \bar{\epsilon}_{\ell_s}\right) dr'$$

$$+ \rho_{\ell_s} \left[\alpha_{\ell_s}(2R\dot{R}^2 + R^2\ddot{R}) + \frac{\rho_{\ell_p}}{\rho_{\ell_s}} \alpha_p (2b\dot{b}^2 + b^2\ddot{b})\right]\left(\frac{1}{a} - \frac{1}{R}\right)$$

$$- \frac{1}{2} \rho_{\ell_s} \left[\left(1 - \frac{\rho_{\ell_p}}{\rho_{\ell_s}}\right) R^2\dot{R} + \frac{\rho_{\ell_p}}{\rho_{\ell_s}} \alpha_p b^2 \dot{b}\right]^2 \left(\frac{1}{a^4} - \frac{1}{R^4}\right)$$

$$+ P_I + \alpha_p \rho_o \dot{b}^2 + \alpha_p \rho_{\ell_p} (2b\dot{b}^2 + b^2\ddot{b}) \left(\frac{1}{R} - \frac{1}{b}\right)$$

$$+ \frac{1}{2} \alpha_p^2 \rho_{\ell_p} b^4 \dot{b}^2 \left(\frac{1}{R^4} - \frac{1}{b^4}\right)$$

$$= \rho_{\ell_p} \left(- \alpha_{\ell_s} \dot{R}^2 + 2\alpha_{\ell_s}\alpha_p \frac{b^2 \dot{b}\dot{R}}{R^2} - \alpha_p^2 \alpha_{\ell_s} \frac{b^4 \dot{b}^2}{R^4}\right) \qquad \text{(III-51)}$$

where

$$P_I = - \frac{2}{3} Y \ln \left(1 - e^{-3\beta} \frac{\rho_o}{\rho_{\ell_p}}\right) + \frac{2}{27} \pi^2 E_t$$

$$- \frac{4}{9} E_t \sum_{n=1}^{\infty} \frac{1}{n^2} \left(1 - e^{-3\beta} \frac{\rho_o}{\rho_{\ell_p}}\right) + \frac{4}{9} E \left(1 - e^{-3\beta}\right) \qquad \text{(III-52)}$$

Solving Eq. III-51 for the dynamic pressure variation at the cavity surface and subsequent rearrangement yields

$$
\begin{aligned}
p(t) \;=\; & 2Y_s \ln\frac{R}{a} + 2\int_a^R \frac{1}{r'} E_{t_s}\left(2\ln\frac{r'}{r_o} + \frac{2}{3}\bar{\epsilon}_{\ell_s}\right)dr' \\
& + \alpha_{\ell_s}\rho_{\ell_s}(2R\dot{R}^2 + R^2\ddot{R})\left(\frac{1}{a} - \frac{1}{R}\right) \\
& + \alpha_p\rho_{\ell_p}(2b\dot{b}^2 + b^2\ddot{b})\left(\frac{1}{a} - \frac{1}{b}\right) \\
& - \frac{1}{2}\rho_{\ell_s}\left[\alpha_{\ell_s}R^2\dot{R} + \frac{\rho_{\ell_p}}{\rho_{\ell_s}}\alpha_p b^2\dot{b}\right]^2\left(\frac{1}{a^4} - \frac{1}{R^4}\right) \\
& + \frac{1}{2}\alpha_p^2\rho_{\ell_p}b^4\dot{b}^2\left(\frac{1}{R^4} - \frac{1}{b^4}\right) + P_I \\
& + \alpha_p\rho_o\dot{b}^2 + \alpha_{\ell_s}\rho_{\ell_p}\dot{R}^2 - 2\rho_{\ell_p}\alpha_{\ell_s}\alpha_p\frac{b^2\dot{b}}{R^2}\dot{R} \\
& + \alpha_p^2\alpha_{\ell_s}\rho_{\ell_p}\frac{b^4\dot{b}^2}{R^4}
\end{aligned}
\qquad \text{(III-53)}
$$

The following relationships have been derived in Ref. 3 and are listed for convenience.

$$
2\int_a^R \frac{1}{r'} E_{t_s}\left(2\ln\frac{r'}{r_o} + \frac{2}{3}\bar{\epsilon}_{\ell_s}\right)dr' = \frac{2}{27}\pi^2 E_{t_s} - \frac{4}{9}E_{t_s}\eta \qquad \text{(III-54)}
$$

$$
2Y_s \ln\frac{R}{a} = -\frac{2}{3}Y_s \ln\left(1 - e^{-3\beta_s}\frac{\rho_{o_s}}{\rho_{\ell_s}}\right) \qquad \text{(III-55)}
$$

where

$$\beta_s = \frac{Y_s}{2E_s} - \frac{\bar{\epsilon}_{\ell_s}}{3} \tag{III-56}$$

$$\alpha_{\ell_s}(2R\dot{R}^2 + R^2\ddot{R}) = 2a\dot{a}^2 + a^2\ddot{a} \tag{III-57}$$

$$\alpha_{\ell_s} R^2\dot{R} = a^2\dot{a} \tag{III-58}$$

$$\frac{b}{R} = \left(1 - e^{-3\beta}\frac{\rho_o}{\rho_{\ell_p}}\right)^{-1/3} \tag{III-59}$$

where

$$\beta = \frac{Y}{2E} - \frac{\bar{\epsilon}_{\ell_p}}{3} \tag{III-60}$$

$$\frac{R}{a} = \left(1 - e^{-3\beta_s}\frac{\rho_{o_s}}{\rho_{\ell_p}}\right)^{-1/3} \tag{III-61}$$

$$(b^3 - R^3)\rho_{\ell_p} = \left(b_o^3 - R_o^3\right)\rho_o \tag{III-62}$$

$$b_o = b\, e^{-\beta} \tag{III-63}$$

Using Eqs. III-62, III-63

$$b^3\left(1 - e^{-3\beta}\frac{\rho_o}{\rho_{\ell_p}}\right) = R^3\left(1 - \frac{R_o^3}{R^3}\frac{\rho_o}{\rho_{\ell_p}}\right) \tag{III-64}$$

or

$$b = R\left(1 - e^{-3\beta}\frac{\rho_o}{\rho_{\ell_p}}\right)^{-1/3}\left(1 - \frac{R_o^3}{R^3}\frac{\rho_o}{\rho_{\ell_p}}\right)^{1/3} \tag{III-65}$$

Now $\rho_{\ell_p} > \rho_o$, and for deep penetration depths of the order of several projectile diameters in ice, $R^3 \gg R_o^3$. Thus, Eq. III-65 becomes

$$b \cong R\bar{\alpha}_p^{-1/3} \tag{III-66}$$

where $$\bar{\alpha}_p = \left(1 - e^{-3\beta}\frac{\rho_o}{\rho_{\ell_p}}\right) \tag{III-67}$$

It follows readily that

$$b = a\left(1 - e^{-3\beta_s}\frac{\rho_{o_s}}{\rho_{\ell_p}}\right)^{-1/3}\left(1 - e^{-3\beta}\frac{\rho_o}{\rho_{\ell_p}}\right)^{-1/3} \tag{III-68}$$

or $$b = a\bar{\alpha}_s^{-1/3}\bar{\alpha}_p^{-1/3} \tag{III-69}$$

where $$\bar{\alpha}_s = \left(1 - e^{-3\beta_s}\frac{\rho_{o_s}}{\rho_{\ell_p}}\right) \tag{III-70}$$

Introduction of these relationships in Eq. III-53 yields

$$p(t) = -\frac{2}{3} Y_s \ln\left(1 - e^{-3\beta_s} \frac{\rho_{o_s}}{\rho_{\ell_s}}\right) + \frac{2}{27} \pi^2 E_{t_s}$$

$$- \frac{4}{9} E_{t_s}\eta + \rho_{\ell_s}(a^2\ddot{a} + 2a\dot{a}^2) \left(\frac{1}{a} - \frac{1}{R}\right)$$

$$+ \alpha_p \rho_{\ell_p} (\bar{\alpha}_p)^{-1} (\bar{\alpha}_s)^{-1} (a^2\ddot{a} + 2a\dot{a}^2) \left(\frac{1}{a} - \frac{1}{b}\right)$$

$$- \frac{1}{2} \rho_{\ell_s} \left[a^2\dot{a} + \frac{\rho_{\ell p}}{\rho_{\ell_s}} \alpha_p (\bar{\alpha}_p)^{-1} (\bar{\alpha}_s)^{-1} a^2 \dot{a}\right]^2 \left(\frac{1}{a^4} - \frac{1}{R^4}\right)$$

$$+ \frac{1}{2} \alpha_p^2 \rho_{\ell p} (\bar{\alpha}_p)^{-2} (\bar{\alpha}_s)^{-2} a^4 \dot{a}^2 \left(\frac{1}{R^4} - \frac{1}{b^4}\right)$$

$$+ P_I + \alpha_p \rho_o (\bar{\alpha}_p)^{-2/3} (\bar{\alpha}_s)^{-2/3} \dot{a}^2 + \rho_{\ell p} \alpha_{\ell_s} (\bar{\alpha}_s)^{-2/3} \dot{a}^2$$

$$- 2\rho_{\ell_p} \alpha_{\ell_s} \alpha_p (\bar{\alpha}_p)^{-1} (\bar{\alpha}_s)^{-2/3} \dot{a}^2$$

$$+ \rho_{\ell_p} \alpha_p^2 \alpha_{\ell_s} (\bar{\alpha}_p)^{-2} (\bar{\alpha}_s)^{-2/3} \dot{a}^2 \qquad \text{(III-71)}$$

Now let

$$P_{II} = P_I - \frac{2}{3} Y_s \ln\left(1 - e^{-3\beta_s} \frac{\rho_{o_s}}{\rho_{\ell_s}}\right)$$

$$+ \frac{2}{27} \pi^2 E_{t_s} - \frac{4}{9} E_{t_s}\eta \qquad \text{(III-72)}$$

Then

$$p(t) = P_{II} + (a\ddot{a} + 2\dot{a}^2)\left[\rho_{\ell_s}\left(1 - \bar{\alpha}_s^{\,1/3}\right) + \frac{\alpha_p \rho_{\ell_p}}{\bar{\alpha}_p \bar{\alpha}_s}\left(1 - \bar{\alpha}_s^{\,1/3}\bar{\alpha}_p^{\,1/3}\right)\right]$$

$$- \frac{1}{2}\rho_{\ell_s}\dot{a}^2\left[1 + \frac{\rho_{\ell_p}}{\rho_{\ell_s}}\frac{\alpha_p}{\bar{\alpha}_p\bar{\alpha}_s}\right]^2\left(1 - \bar{\alpha}_s^{\,4/3}\right)$$

$$+ \frac{1}{2}\frac{\alpha_p^{\,2}\rho_{\ell_p}}{\bar{\alpha}_p^{\,2}\bar{\alpha}_s^{\,2}}\left(1 - \bar{\alpha}_s^{\,4/3}\right)\dot{a}^2$$

$$+ \dot{a}^2\left[\frac{\alpha_p \rho_o}{\bar{\alpha}_p^{\,2/3}\bar{\alpha}_s^{\,2/3}} + \frac{\alpha_{\ell_s}\rho_{\ell_p}}{\bar{\alpha}_s^{\,2/3}} - 2\rho_{\ell_p}\frac{\alpha_{\ell_s}\alpha_p}{\bar{\alpha}_p\bar{\alpha}_s^{\,2/3}} + \frac{\alpha_p^{\,2}\alpha_{\ell_s}}{\bar{\alpha}_p^{\,2}\bar{\alpha}_s^{\,2/3}}\rho_{\ell_p}\right] \qquad \text{(III-73)}$$

Rearranging

$$p(t) = P_{II} + a\ddot{a}\left[\rho_{\ell_s}\left(1 - \bar{\alpha}_s^{\,1/3}\right) + \frac{\alpha_p\rho_{\ell_p}}{\bar{\alpha}_p\bar{\alpha}_s}\left(1 - \bar{\alpha}_s^{\,1/3}\bar{\alpha}_p^{\,1/3}\right)\right]$$

$$+ \dot{a}^2\left[2\rho_{\ell_s}\left(1 - \bar{\alpha}_s^{\,1/3}\right) + \frac{2\alpha_p\rho_{\ell_p}}{\bar{\alpha}_p\bar{\alpha}_s}\left(1 - \bar{\alpha}_s^{\,1/3}\bar{\alpha}_p^{\,1/3}\right)\right.$$

$$- \frac{1}{2}\rho_{\ell_s}\left(1 + \frac{\rho_{\ell_p}}{\rho_{\ell_s}}\frac{\alpha_p}{\bar{\alpha}_p\bar{\alpha}_s}\right)^2\left(1 - \bar{\alpha}_s^{\,4/3}\right)$$

$$+ \frac{1}{2}\frac{\alpha_p^{\,2}\rho_{\ell_p}}{\bar{\alpha}_p^{\,2}\bar{\alpha}_s^{\,2}}\left(1 - \bar{\alpha}_s^{\,4/3}\right) + \frac{\alpha_p\rho_o}{\bar{\alpha}_p^{\,2/3}\bar{\alpha}_s^{\,2/3}} + \frac{\alpha_{\ell_s}\rho_{\ell_p}}{\bar{\alpha}_s^{\,2/3}}$$

$$\left.- 2\rho_{\ell_p}\frac{\alpha_{\ell_s}\alpha_p}{\bar{\alpha}_p\bar{\alpha}_s^{\,2/3}} + \frac{\alpha_p^{\,2}\alpha_{\ell_s}}{\bar{\alpha}_p^{\,2}\bar{\alpha}_s^{\,2/3}}\rho_{\ell_p}\right] \qquad \text{(III-74)}$$

Equation III-74 can be rewritten as

$$p(t) = P_{II} + C_1 a\ddot{a} + C_2 \dot{a}^2 \tag{III-75}$$

where

$$C_1 = \rho_{\ell_s}\left(1 - \alpha_s^{1/3}\right) + \frac{\alpha_p \rho_{\ell_p}}{\bar{\alpha}_p \bar{\alpha}_s}\left(1 - \bar{\alpha}_s^{1/3} \bar{\alpha}_p^{1/3}\right) \tag{III-76}$$

$$\begin{aligned} C_2 = {} & 2\rho_{\ell_s}\left(1 - \bar{\alpha}_s^{1/3}\right) + 2\,\frac{\alpha_p \rho_{\ell_p}}{\bar{\alpha}_s \bar{\alpha}_p}\left(1 - \bar{\alpha}_s^{1/3} \bar{\alpha}_p^{1/3}\right) \\ & - \frac{1}{2}\rho_{\ell_s}\left(1 + \frac{\rho_{\ell_p}}{\rho_{\ell_s}}\,\frac{\alpha_p}{\bar{\alpha}_p \bar{\alpha}_s}\right)^2 \left(1 - \bar{\alpha}_s^{4/3}\right) \\ & + \frac{1}{2}\,\frac{\rho_{\ell_p} \alpha_p^2}{\bar{\alpha}_p^2 \bar{\alpha}_s^2}\left(1 - \bar{\alpha}_s^{4/3}\right) + \frac{\alpha_p \rho_o}{\bar{\alpha}_p^{2/3} \bar{\alpha}_s^{2/3}} + \frac{\alpha_{\ell_s} \rho_{\ell_p}}{\bar{\alpha}_s^{2/3}} \\ & - 2\rho_{\ell_p}\,\frac{\alpha_{\ell_s} \alpha_p}{\bar{\alpha}_p \bar{\alpha}_s^{2/3}} + \frac{\alpha_p^2 \alpha_{\ell_s}}{\bar{\alpha}_p^2 \bar{\alpha}_s^{2/3}}\,\rho_{\ell_p} \end{aligned} \tag{III-77}$$

It has been shown in Ref. 3 that an expression for dynamic pressure at the spherical cavity surface, such as the one given by Eq. III-75, can be employed to derive an equation of motion for projectile penetration. In the present case, the following relationship is obtained

$$M\ddot{q} = -\left[P_{II} + \frac{2}{3}\left(C_1 \frac{D}{2}\ddot{q} + C_2 \dot{q}^2\right)\right]\frac{\pi D^2}{4} \tag{III-78}$$

The usual process of integration yields

$$\ln \frac{P_{II} + 2/3\ C_1 \dot{q}^2}{P_{II} + 2/3\ C_2 v_{o_s}^2} = -\frac{4}{3} \frac{C_2 \left(q - q_{o_s}\right)}{M/A + C_1 D/3} \tag{III-79}$$

Solving for the terminal penetration depth

$$q_t = q_{o_s} + \frac{3}{4} \frac{M/A + C_1 D/3}{C_2} \ln \frac{P_{II} + 2/3\ C_2 v_{o_s}^2}{P_{II}} \tag{III-80}$$

Here, q_{o_s} and v_{o_s} refer to the penetration depth and exit velocity, respectively, characterizing projectile transit through the overlying snow cover. These quantities are obtained from solution of the first phase penetration problem.

Appendix A

LISTING FOR LARGE DEFORMATION THEORY COMPUTER CODE THAT INCORPORATES STRESS REDISTRIBUTIONS DUE TO FRACTURE

Preceding page blank

JOB 485

```
//LISTER  JOB  'A519***,602,1,9','SIDHU'
//*SERVICE  LIST
//*PRINT COPIES=2
//STEP1  EXEC FORTHCLG
//FORT.SYSIN  DD *
C ***********************************************************************
C
C     VERSION FSP1....TWO DIMENSIONAL AXISYMMETRIC 'CANDIA' CODE.
C
C     THIS VERSION COMPATIBLE ON ANY IBM 360 MACHINE
C
C     LANGUAGE....FORTRAN (IV) H
C
C
C
C ***********************************************************************
      IMPLICIT REAL*8 (A-H,O-Z,$), INTEGER (I-N)
      COMMON /CCM1/ AR   (24,46),  ARH (24,46),    ARDH(24,46),
     1              AZ   (24,46),  AZH (24,46),    AZDH(24,46),
     2              AA   (24,46),  AV  (24,46),    AP  (24,46),
     3              ATRR(24,46),   ATZZ(24,46),    ATRZ(24,46),
     4              ATTT(24,46),
     5              ASRR(24,46),   ASZZ(24,46),    ASRZ(24,46),
     6              ASTT(24,46)
      COMMON /CCM2/ BR   (46, 6),  BRH (46, 6),    BRDH(46, 6),
     1              BZ   (46, 6),  BZH (46, 6),    BZDH(46, 6),
     2              BA   (46, 6),  BV  (46, 6),    BP(46, 6),
     3              BTRR(46, 6),   BTZZ(46, 6),    BTRZ(46, 6),
     4              BTTT(46, 6),
     5              BSRR(46, 6),   BSZZ(46, 6),    BSRZ(46, 6),
     6              BSTT(46, 6)
      COMMON /CCM3/ TR   (24,46),  TRH (24,46),    TRDH(24,46),
     1              TZ   (24,46),  TZH (24,46),    TZDH(24,46),
     2   SR(46),  SZ(46),  TSR(46),  TSZ(46),  ZM(2,46),  SPTM(5),
     4 A1(2), A2(2), A3(2), A4(2), ALEN(2), AMAT(2), AMU(2), ANU(2),
     5 BK(2), CAPE(2), CDOT(2), DR(2), DZ(2), RAD(2), RHO(2), RDOT(2),
     6 RHOG(2), RHCT(2), TWMU(2), YT(2), ZDOT(2), ZV(2), VCON(2),
     7 CQ,DTN,CTH,DTHN,DTMIN,ZDINT,TYME,CHEKD,FACTR
      COMMON /ICCM1/  IPTA(24,46)
      COMMON /ICCM2/ IPTB(46, 6)
      COMMON /ICCM3/ ISHP(2), JM(2), KM(2),      KMID,
     1 IRUN,ICT1,ICT2,IDENT,IMAX,IPRNT,ICYCL,IMAP,NTT,KINT,KMN,KMX
C *****
C
C     SET ARTIFICIAL VISCOSITY COEFFICIENT VALUE
C
C *****
      CQ=C.0
      READ (5,100) IDENT
  100 FORMAT (I10)
C *****
C
C   IDENT = 1  NEW RUN NO SAVE
C         = 2  NEW RUN SAVE
C         = 3  RESUME NO SAVE
C         = 4 RESUME SAVE
C
C *****
      IF (IDENT.GE.3) CALL RSME
      IF (IDENT.GE.3) GO TO 302
```

```
C *****
C
C     READ IN DATA FROM CARDS
C
C *****
      CALL INPTT
C *****
C
C     DO ALL INITIAL MODIFICATION OF DATA...UNITS ETC
C
C *****
      CALL INITT
C *****
C
C     CREATE THE FINITE DIFFERENCE GRID WORK AND ALL DISC FILES
C
C *****
      CALL GRIDD
C *****
C
C     WRITE OUT THE SALIENT ASPECTS OF THE PROBLEM
C
C *****
  302 CALL INWRR
C *****
C
C     CYCLE THROUGH THE BODY IS INITIATED
C
C *****
  300 CALL LETGC
C *****
C
C     DO ALL NECESSARY COMPUTATION FOR THE TARGET
C
C *****
      CALL       GUTT ( 1,IPTA,AR,AZ,ARH,AZH,ARDH,AZDH,ATRR,ATZZ,ATRZ,
     1                 ATTT,ASRR,ASZZ,ASRZ,ASTT,AA,AV,AP,KMID,KMN,KINT,
     2                 JMX)
C *****
C
C     BEFORE GOING TO THE PROJECTILE DO ALL PREPARATIONS NECESSARY
C
C *****
      JJJ=JM(2)
      DO 304 J=1,JJJ
      BRH(1,J) = ARH(KINT,J)
      BZH(1,J) = AZH(KINT,J)
      BRDH(1,J) = ARDH(KINT,J)
  304 BZDH(1,J) = AZDH(KINT,J)
C *****
C
C     NOW DO ALL THE COMPUTATIONS FOR THE PROJECTILE
C
C *****
      K2 = KM(2)
      CALL       GUTT ( 2,IPTB,BR,BZ,BRH,BZH,BRDH,BZDH,BTRR,BTZZ,BTRZ,
     1                 BTTT,BSRR,BSZZ,BSRZ,BSTT,BA,BV,BP,K2, 2,KMX,
     2                 JM(2)  )
C *****
C
```

```
C     CHECK IF IT IS TIME TO QUIT
C
C *****
      IF (ICYCL-IMAX) 300,301,301
  301 IF ((IDENT.EQ.2).OR.(IDENT.EQ.4)) CALL SSAV
      RETURN
      END
      SUBROUTINE RSNF
      IMPLICIT REAL*8 (A-H,O-Z,$),  INTEGER (I-N)
      COMMON /CCM1/ AR   (24,46),  ARH (24,46),   ARDH(24,46),
     1              AZ   (24,46),  AZH (24,46),   AZDH(24,46),
     2              AA   (24,46),  AV  (24,46),   AP  (24,46),
     3              ATRR(24,46),  ATZZ(24,46),   ATRZ(24,46),
     4              ATTT(24,46),
     5              ASRR(24,46),  ASZZ(24,46),   ASRZ(24,46),
     6              ASTT(24,46)
      COMMON /CCM2/ BR   (46, 6),  BRH (46, 6),   BRDH(46, 6),
     1              BZ   (46, 6),  BZH (46, 6),   BZDH(46, 6),
     2              BA   (46, 6),  BV  (46, 6),   BP(46, 6),
     3              BTRR(46, 6),  BTZZ(46, 6),   BTRZ(46, 6),
     4              BTTT(46, 6),
     5              BSRR(46, 6),  BSZZ(46, 6),   BSRZ(46, 6),
     6              BSTT(46, 6)
      COMMON /CCM3/ TR   (24,46),  TRH (24,46),   TRDH(24,46),
     1              TZ   (24,46),  TZH (24,46),   TZDH(24,46),
     2    SR(46),  SZ(46),  TSR(46),  TSZ(46),  ZM(2,46),  SPTM(5),
     4 A1(2), A2(2), A3(2), A4(2), ALEN(2), AMAT(2), AMU(2), ANU(2),
     5 BK(2), CAPE(2), CDOT(2), DR(2), DZ(2), RAD(2), RHO(2), RDOT(2),
     6 RHOG(2), RHOT(2), TWMU(2), YT(2), ZDOT(2), ZV(2), VCON(2),
     7 CQ,DTN,CTH,DTHW,DTMIN,ZOINT,TYME,CHEKD,FACTR
      COMMON /ICCM1/  IPTA(24,46)
      COMMON /ICOM2/ IPTB(46, 6)
      COMMON /ICCM3/ ISHP(2), JM(2), KM(2),      KMID,
     1 IRUN,ICT1,IDT2,IDENT,IMAX,IPRNT,ICYCL,IMAP,NTT,KINT,KMN,KMX
      RETURN
      END
      SUBROUTINE SSAV
      IMPLICIT REAL*8 (A-H,O-Z,$),  INTEGER (I-N)
      COMMON /CCM1/ AR   (24,46),  ARH (24,46),   ARDH(24,46),
     1              AZ   (24,46),  AZH (24,46),   AZDH(24,46),
     2              AA   (24,46),  AV  (24,46),   AP  (24,46),
     3              ATRR(24,46),  ATZZ(24,46),   ATRZ(24,46),
     4              ATTT(24,46),
     5              ASRR(24,46),  ASZZ(24,46),   ASRZ(24,46),
     6              ASTT(24,46)
      COMMON /CCM2/ BR   (46, 6),  BRH (46, 6),   BRDH(46, 6),
     1              BZ   (46, 6),  BZH (46, 6),   BZDH(46, 6),
     2              BA   (46, 6),  BV  (46, 6),   BP(46, 6),
     3              BTRR(46, 6),  BTZZ(46, 6),   BTRZ(46, 6),
     4              BTTT(46, 6),
     5              BSRR(46, 6),  BSZZ(46, 6),   BSRZ(46, 6),
     6              BSTT(46, 6)
      COMMON /CCM3/ TR   (24,46),  TRH (24,46),   TRDH(24,46),
     1              TZ   (24,46),  TZH (24,46),   TZDH(24,46),
     2    SR(46),  SZ(46),  TSR(46),  TSZ(46),  ZM(2,46),  SPTM(5),
     4 A1(2), A2(2), A3(2), A4(2), ALEN(2), AMAT(2), AMU(2), ANU(2),
     5 BK(2), CAPE(2), CDOT(2), DR(2), DZ(2), RAD(2), RHO(2), RDOT(2),
     6 RHOG(2), RHOT(2), TWMU(2), YT(2), ZDOT(2), ZV(2), VCON(2),
     7 CQ,DTN,CTH,DTHW,DTMIN,ZOINT,TYME,CHEKD,FACTR
      COMMON /ICCM1/  IPTA(24,46)
```

```
      COMMON /ICOM2/ IPTB(46, 6)
      COMMON /ICCM3/ ISHP(2), JM(2), KM(2),      KMID,
     1 IRUN,IDT1,IDT2,IDENT,IMAX,IPRNT,ICYCL,IMAP,NTT,KINT,KMN,KMX
      RETURN
      END
      SUBROUTINE INPTT
      IMPLICIT REAL*8 (A-H,O-Z,$), INTEGER (I-N)
      COMMON /CCM1/ AR   (24,46),  ARH (24,46),    ARDH(24,46),
     1              AZ   (24,46),  AZH (24,46),    AZDH(24,46),
     2              AA   (24,46),  AV   (24,46),   AP   (24,46),
     3              ATRR(24,46),   ATZZ(24,46),    ATRZ(24,46),
     4              ATTT(24,46),
     5              ASRR(24,46),   ASZZ(24,46),    ASRZ(24,46),
     6              ASTT(24,46)
      COMMON /CCM2/ BR   (46, 6),  BRH (46, 6),    BRDH(46, 6),
     1              BZ   (46, 6),  BZH (46, 6),    BZDH(46, 6),
     2              BA   (46, 6),  BV   (46, 6),   BP(46, 6),
     3              BTRR(46, 6),   BTZZ(46, 6),    BTRZ(46, 6),
     4              BTTT(46, 6),
     5              BSRR(46, 6),   BSZZ(46, 6),    BSRZ(46, 6),
     6              BSTT(46, 6)
      COMMON /CCM3/ TR   (24,46),  TRH (24,46),    TRDH(24,46),
     1              TZ   (24,46),  TZH (24,46),    TZDH(24,46),
     2    SR(46),   SZ(46),   TSR(46),   TSZ(46),   ZM(2,46),   SPTM(5),
     4 A1(2), A2(2), A3(2), A4(2), ALEN(2), AMAT(2), AMU(2), ANU(2),
     5 BK(2), CAPF(2), CDOT(2), DR(2), DZ(2), RAD(2), RHO(2), RDOT(2),
     6 RHOG(2), RHOT(2), TWMU(2), YT(2), ZDOT(2), ZV(2), VCON(2),
     7 CQ,DTN,DTH,DTHW,DTMIN,ZDINT,TYME,CHEKD,FACTR
      COMMON /ICCM1/  IPTA(24,46)
      COMMON /ICCM2/ IPTB(46, 6)
      COMMON /ICCM3/ ISHP(2), JM(2), KM(2),      KMID,
     1 IRUN,IDT1,IDT2,IDENT,IMAX,IPRNT,ICYCL,IMAP,NTT,KINT,KMN,KMX
      NCR=5
C ******
C
C  READ IDENTIFIERS:
C
C      IRUN : RUN I.D. NO
C      IDT1 : MONTH-DAY   (EG. 1103 - 3RD NOV)
C      IDT2 : YEAR        (EG. 69   - FOR 1969)
C      FACTR : FRACTION OF DIAGONAL USED TO FIND THE TIME STEP
C
C ******
      READ (NCR,100) IRUN,IDT1,IDT2,FACTR
C ******
C
C  READ CONTROL VARIABLES
C
C      IMAX - MAXIMUM NO. OF CYCLES BEFORE PROBLEM TERMINATION
C      IPRNT- MAXIMUM NO. OF CYCLES BETWEEN PRINTOUTS
C      IMAP - MAXIMUM NO. OF CYCLES BETWEEN GRAPHICAL OUTPUTS
C      NTT  - NO. OF SPECIAL PRINT TIMES
C      SPTM(NTT)  SPECIAL PRINT TIMES  IN MICRO-SECONDS
C
C ******
      READ (NCR,101) IMAX,IPRNT,IMAP,NTT
      READ (NCR,102) (SPTM(K), K=1,NTT)
C ******
C
C  READ MATERIAL PROPERTIES
```

```
C
C     AMAT(2) - MATERIAL NAME (4 CHARACTERS MAXIMUM)
C     ANU(2)  - POISSON'S RATIO
C     CAPE(2) - YOUNG'S MODULUS - PSI
C     RHO(2)  - DENSITY - GM/CC
C     YT(2)   - TENSILE YIELD STRENGTH - PSI
C     A1-A4(2)- SPARES
C
C ******
      DO 300 J=1,2
      READ (NCR,103) AMAT(J),ANU(J),CAPE(J),RHO(J),YT(J)
      READ (NCR,104) A1(J),A2(J),A3(J),A4(J)
C ******
C
C  READ GEOMETRIC PROPERTIES
C     ISHP(2) - SHAPE FACTOR
C          BODY 1 (TARGET) = 1 ,ROD
C                            2 ,PLATE
C          BODY 2 (PROJECTILE)=1 ,ROD
C                            2 ,PLATE
C     RDOT(2) - INITIAL VELOCITY IN R DIRECTION - FT/SEC
C     ZDOT(2) - INITIAL VELOCITY IN Z DIRECTION - FT/SEC
C     JM(2)   - NO. OF CELLS IN R DIRECTION
C     KM(2)   - NO. OF CELLS IN Z DIRECTION
C     DR(2)   - SIZE OF CELLS IN R DIRECTION - INCH
C     DZ(2)   - SIZE OF CELLS IN Z DIRECTION - INCH
C
C ******
  300 READ (NCR,105) ISHP(J),RDOT(J),ZDOT(J),JM(J),KM(J),DR(J),DZ(J)
  100 FORMAT (3I10,F10.5)
  101 FORMAT (4I10)
  102 FORMAT (8F10.0)
  103 FORMAT (6X,A4,4F10.0)
  104 FORMAT (4F10.0)
  105 FORMAT (I10,2F10.0,2I10,2F10.0)
      RETURN
      END
      SUBROUTINE INITT
C *****
C  THIS SUBROUTINE PREPARES THE INPUT DATA FOR PRINTING  AND SUBSEQUENT
C  USE
C
C *****
      IMPLICIT REAL*8 (A-H,O-Z,$), INTEGER (I-N)
      COMMON /CCM1/ AR  (24,46),  ARH (24,46),   ARDH(24,46),
     1              AZ  (24,46),  AZH (24,46),   AZDH(24,46),
     2              AA  (24,46),  AV  (24,46),   AP  (24,46),
     3              ATRR(24,46),  ATZZ(24,46),   ATRZ(24,46),
     4              ATTT(24,46),
     5              ASRR(24,46),  ASZZ(24,46),   ASRZ(24,46),
     6              ASTT(24,46)
      COMMON /CCM2/ BR  (46, 6),  BRH (46, 6),   BRDH(46, 6),
     1              BZ  (46, 6),  BZH (46, 6),   BZDH(46, 6),
     2              BA  (46, 6),  BV  (46, 6),   BP(46, 6),
     3              BTRR(46, 6),  BTZZ(46, 6),   BTRZ(46, 6),
     4              BTTT(46, 6),
     5              BSRR(46, 6),  BSZZ(46, 6),   BSRZ(46, 6),
     6              BSTT(46, 6)
      COMMON /CCM3/ TR  (24,46),  TRH (24,46),   TRDH(24,46),
     1              TZ  (24,46),  TZH (24,46),   TZDH(24,46),
```

```
     2   SR(46),   SZ(46),   TSR(46),   TSZ(46),   ZM(2,46),   SPTM(5),
     4 A1(2), A2(2), A3(2), A4(2), ALEN(2), AMAT(2), AMU(2), ANU(2),
     5 BK(2), CAPE(2), CDOT(2), DR(2), DZ(2), RAD(2), RHO(2), RDOT(2),
     6 RHOG(2), RHOT(2), TWMU(2), YT(2), ZDOT(2), ZV(2), VCON(2),
     7 CQ,DTN,CTH,DTHW,DTMIN,ZDINT,TYME,CHEKD,FACTR
      COMMON /ICOM1/  IPTA(24,46)
      COMMON /ICOM2/  IPTB(46, 6)
      COMMON /ICOM3/  ISHP(2), JM(2), KM(2),       KMID,
     1 IRUN,IDT1,IDT2,IDENT,IMAX,IPRNT,ICYCL,IMAP,NTT,KINT,KMN,KMX
      DO 301   J=1,2
      RHOG(J)= RHO(J)
      RHO(J) = 9.35482D-05 * RHO(J)
C *****
C
C NOW RHOG IS IN   (GM/CC)
C     RHO  IS IN   (LB-SEC2/IN4)
C
C *****
      AMU(J)= (CAPE(J)*0.5)/(1.0+ANU(J))
      BK(J) =   CAPE(J)/(3.0*(1.0-2.0*ANU(J)))
      CDOT(J)=DSQRT((BK(J)+4.0*AMU(J)/3.0)/RHO(J))*1.0D-06
      VCON(J)= CDOT(J)/1.2D-05
C *****
C
C  UNITS:   AMU,BK ARE IN (PSI)
C           CDOT    IS   IN (INCHES PER MICROSEC)
C           VCON    IS   IN (FEET PER SEC)
C
C *****
      ALEN(J) = DZ(J) * KM(J)
      RAD(J)  = DR(J) * JM(J)
C *****
C  NOW CHANGE   JM AND KM TO STAND FOR NO OF GRID ROWS AND COLUMNS
C                            INSTEAD OF NO OF GRID CELLS
C *****
      JM(J) = JM(J) + 1
      KM(J) = KM(J) + 1
      ZV(J) = ZDOT(J)
      ZDOT(J) = 12.0D-06 * ZDOT(J)
C *****
C  UNITS:   ZV IS   IN (FT/SEC)
C           ZDOT IS IN (INCH PER MICROSEC)
C *****
C
C
C NEXT COMPUTE VALUES FOR TIME STEP IN MICROSEC
C
      DTHW =FACTR*DSQRT(DR(J)**2 + DZ(J)**2) / CDOT(J)
C *****
      IF (J-1) 300,300,301
  300 DTH=DTHW
  301 CONTINUE
C *****
C  NOW SELECT THE SMALLER OF DTH & DTHW TO BE THE TIME STEP
C *****
      IF (DTH-DTHW) 303,303,302
  302 DTH=DTHW
  303 DTMIN = DTH * 0.1
      KINT = KM(1)
      KMID = KINT + 1
```

```
C *****
C--   COMPUTE THE INTERFACE VELOCITY
C *****
      ZDINT= (ZDOT(1)*RHO(1)*CDOT(1) + ZDOT(2)*RHO(2)*CDOT(2))/(RHO(1)*
     1          CDOT(1) + RHO(2)*CDOT(2))
      RETURN
      END
      SUBROUTINE INWRR
      IMPLICIT REAL*8 (A-H,O-Z,$), INTEGER (I-N)
      COMMON /COM1/ AR   (24,46),   ARH (24,46),    ARDH(24,46),
     1              AZ   (24,46),   AZH (24,46),    AZDH(24,46),
     2              AA   (24,46),   AV   (24,46),   AP   (24,46),
     3              ATRR(24,46),    ATZZ(24,46),    ATRZ(24,46),
     4              ATTT(24,46),
     5              ASRR(24,46),    ASZZ(24,46),    ASRZ(24,46),
     6              ASTT(24,46)
      COMMON /COM2/ BR   (46, 6),   BRH (46, 6),    BRDH(46, 6),
     1              BZ   (46, 6),   BZH (46, 6),    BZDH(46, 6),
     2              BA   (46, 6),   BV   (46, 6),   BP(46, 6),
     3              BTRR(46, 6),    BTZZ(46, 6),    BTRZ(46, 6),
     4              BTTT(46, 6),
     5              BSRR(46, 6),    BSZZ(46, 6),    BSRZ(46, 6),
     6              BSTT(46, 6)
      COMMON /COM3/ TR   (24,46),   TRH (24,46),    TRDH(24,46),
     1              TZ   (24,46),   TZH (24,46),    TZDH(24,46),
     2   SR(46),  SZ(46),   TSR(46),   TSZ(46),   ZM(2,46),   SPTM(5),
     * A1(2), A2(2), A3(2), A4(2), ALEN(2), AMAT(2), AMU(2), ANU(2),
     5 BK(2), CAPE(2), CDOT(2), DR(2), DZ(2), RAD(2), RHO(2), RDOT(2),
     6 RHOG(2), RHOT(2), TWM(2), YT(2), ZDOT(2), ZV(2), VCON(2),
     7 CQ,DTN,DTH,DTHW,DTMIN,ZDINT,TYME,CHEKD,FACTR
      COMMON /ICOM1/  IPTA(24,46)
      COMMON /ICOM2/  IPTB(46, 6)
      COMMON /ICOM3/  ISHP(2), JM(2), KM(2),      KMID,
     1 IRUN,IDT1,IDT2,IDENT,IMAX,IPRNT,ICYCL,IMAP,NTT,KINT,KMN,KMX
      NPR=6
C *****
C
C  WRITE PROBLEM IDENTIFICATION
C
C ******
      WRITE (NPR,200) IRUN,IDT1,IDT2,IDENT
C ******
C
C  WRITE MATERIAL PROPERTIES
C
C ******
      WRITE (NPR,201) AMAT(1),AMAT(2),RHOG(1),RHOG(2),RHO(1),RHO(2)
      WRITE (NPR,202) ANU(1),ANU(2),CAPE(1),CAPE(2),AMU(1),AMU(2)
      WRITE (NPR,203) BK(1),BK(2),CDOT(1),CDOT(2),VCON(1),VCON(2)
      WRITE (NPR,204) YT(1),YT(2),A1(1),A1(2)
C ******
C
C  GEOMETRIC PROPERTIES
C
C ******
      WRITE (NPR,205) ALEN(1),ALEN(2),RAD(1),RAD(2),DR(1),DR(2)
      WRITE (NPR,206) DZ(1),DZ(2),KM(1),KM(2),JM(1),JM(2)
      WRITE (NPR,207) RDOT(1),RDOT(2),ZDOT(1),ZDOT(2),ZV(1),ZV(2)
C ******
C
```

```
C CCNTROL DATA
C
C ******
      WRITE (NPR,208) FACTR,IMAX,IMAP,IPRNT,NTT
      WRITE (NPR,209) (SPTM(KKT),KKT=1,NTT)
C *****
C
C  FCRMATS
C
C *****
200  FORMAT ('1',70X,'TWO-D LAGRANGIAN COMPUTATION',/,71X,'RUN. NO   ',
     S    I4,'  CATE - ',I4,', 19'I2,10X,'RESUME CODE ',I4,
     S                                     ////,40X,'UNITS',29X,'BODY A',14X,
     S    'BODY B',//)
201  FORMAT (2X,'MATERIAL PROPERTIES',//,5X,'MATERIAL',63X,A4,16X,A4,
     S //,5X,'CENSITY (RHO)',22X,'GM/CC',15X,2D20.6,/,40X,'LB-SEC2/IN4',
     S      9X,2C20.6,/)
202  FORMAT (5X,21HPOISSON'S RATIC (ANU), 34X,2D20.6,//,5X,22HYOUNG'S M
     SODULUS (CAPE),13X,'PSI',17X,2D20.6,//,5X,'RIGIDITY MODULUS (AMU)',
     S      13X,'PSI',17X,2D2C.6,/)
203  FORMAT (5X,'BULK MODULUS (BK)',18X,'PSI',17X,2D20.6,//,5X,
     S    'SOUND SPEED (CDOT)',17X,'IN/MICRO-SEC',8X,
     S      2D2C.6,/,40X,'FT/SFC',
     S      14X,2D20.6,/)
204  FORMAT (5X,'YIELD STRENGTH (YT)',16X,'PSI',17X,2D20.6,/
     1          5X,'SHEAR STRENGTH (ST)',16X,'PSI',17X,2D20.6//)
205  FORMAT (2X,'GEOMETRIC PROPFRTIES',//,5X,'LENGTH (ALEN)',22X,'INCH'
     S     ,16X,2D20.6,//,5X,'RADIUS (RAD)',23X,'INCH',16X,2D20.6,//,5X,
     S     'DR',33X,'INCH',16X,2D20.6,/)
206  FORMAT (5X,'DZ',33X,'INCH',16X,2D20.6,//,5X,'GRID POINTS Z (KM)',
     S      37X,2I20   ,//,5X,'GRID POINTS R (JM)',37X,2I20   ,/)
207  FORMAT (5X,'RDCT',31X,'FT/SFC',14X,2D20.6,//,5X,'ZDOT',31X,
     S      'IN/MICRO-SEC',8X,2D2C.6,/,40X,'FT/SEC',14X,2D20.6,///,2X,
     S      'CCNTRCL CATA',/)
208  FORMAT ('C',4X,'TIME STEP FACTOR',18X,F10.5//5X,'MAX NO OF CYCLES'
     1   ,18X,I10//5X,'MAP FREQUENCY',21X,I10//5X,'PRINT FREQUENCY',19X,
     2   I10//5X,'NO OF SPECIAL PRINTS',14X,I10//5X,'SPECIAL PRINT TIMFS
     3 IN MICRC-SEC',5X)
209  FORMAT (5F15.5)
     RETURN
     END
     SUBROUTINE  GRIDD
     IMPLICIT REAL*8 (A-H,O-Z,$), INTEGER (I-N)
     COMMON /CCM1/ AR   (24,46),  ARH (24,46),   ARDH(24,46),
    1              AZ   (24,46),  AZH (24,46),   AZDH(24,46),
    2              AA   (24,46),  AV  (24,46),   AP  (24,46),
    3              ATRR(24,46),   ATZZ(24,46),   ATRZ(24,46),
    4              ATTT(24,46),
    5              ASRR(24,46),   ASZZ(24,46),   ASRZ(24,46),
    6              ASTT(24,46)
     COMMON /CCM2/ BR   (46, 6),  BRH (46, 6),   BRDH(46, 6),
    1              BZ   (46, 6),  BZH (46, 6),   BZDH(46, 6),
    2              BA   (46, 6),  BV  (46, 6),   BP(46, 6),
    3              BTRR(46, 6),   BTZZ(46, 6),   BTRZ(46, 6),
    4              BTTT(46, 6),
    5              BSRR(46, 6),   BSZZ(46, 6),   BSRZ(46, 6),
    6              BSTT(46, 6)
     COMMON /CCM3/ TR   (24,46),  TRH (24,46),   TRDH(24,46),
    1              TZ   (24,46),  TZH (24,46),   TZDH(24,46),
    2    SR(46),  SZ(46),  TSR(46),  TSZ(46),  ZM(2,46),  SPTM(5),
```

```
     4 A1(2), A2(2), A3(2), A4(2), ALFN(2), AMAT(2), AMU(2), ANU(2),
     5 BK(2), CAPE(2), CDOT(2), DR(2), DZ(2), RAD(2), RHO(2), RDOT(2),
     6 RHOG(2), RHOT(2), TWMU(2), YT(2), ZDOT(2), ZV(2), VCON(2),
     7 CQ,DTN,DTH,DTHW,DTMIN,ZDINT,TYMF,CHEKD,FACTR
      COMMON /ICCM1/  IPTA(24,46)
      COMMON /ICCM2/ IPTB(46, 6)
      COMMON /ICOM3/ ISHP(2), JM(2), KM(2),      KMID,
     1 IRUN,ICT1,IDT2,ICENT,IMAX,IPRNT,ICYCL,IMAP,NTT,KINT,KMN,KMX
      DIMENSION AREA(2), ARHO(2)
      NPR=6
      DO 300 I=1,2
C *****
C  I=1  IS THE TARGET
C   =2  IS THE PROJECTILE
C
C      AREA IS THE INITIAL AREA OF EACH CELL IN SQUARE INCHES
C      ARHO IS THE PRODUCT   AREA * DENSITY   IN (LB-MICROSEC-2/IN-2)
C      RHOT IS ONE-THIRD RHO IN LB MICROSEC INCH UNITS
C      TWMU IS TWICE THE RIGIDITY MODULUS AMU.
C
C *****
      AREA(I)= DR(I) * DZ(I)
      ARHO(I)= AREA(I) * RHO(I) * 1.0D12
      RHOT(I)= RHO(I)/3.0D-12
      TWMU(I)= 2.0 * AMU(I)
  300 CONTINUE
C *****
C
C    NOW GRID IS CREATED FOR BODY 1
C
C *****
      ZCUR = 0.0
      JJJ  = JM(1)
      JJ2  = JM(2)
      KKK  = KM(1)
      DO 7777 K=1,KKK
      DO 7777 J=1,JJJ
 7777 IPTA(K,J)=0
      DO 326  K=1,KKK
      RCUR = 0.0
      DO 325  J=1,JJJ
      AR   (K,J) = RCUR
      AZ   (K,J) = ZCUR
      ARH  (K,J) = RCUR
      AZH  (K,J) = ZCUR
      ARDH(K,J) = RDOT(1)
      AZDH(K,J) = ZDOT(1)
      IF (J-1)  304,304,302
  302 IF (K-1)  304,304,303
  303 AA(K,J)=AREA(1)
      AV(K,J)=1.0
      AP(K,J)=0.0
      ATRR(K,J)=0.0
      ATZZ(K,J)=0.0
      ATRZ(K,J)=0.0
      ATTT(K,J)=0.0
      ASRR(K,J)=0.0
      ASZZ(K,J)=0.0
      ASRZ(K,J)=0.0
      ASTT(K,J)=0.0
```

```
C *****
C  NOW THE POINT CONDITION CODE IS DETERMINED
C *****
C *****
C
C
C
C
C POINT CONDITION CODES:
C                                                              8
C                      6             10                       /
C                      *****************************        /
C                      *                            *     7
C                      *                            *  /
C                      *                            */
C                      *                            *   9
C                      *                            *  /
C                      *           REGION 1         */        10          8
C                     5**12                         *********************
C                      *                            *                    *
C                      *                            *     REGION 2       *
C                      *                            *11                  *
C                      *              *             *         *          *7
C                      *              1             *         1          *
C          AXIS -------**********************************************************--------
C                     3               2            13         2          4
C
C
C
C *****
  304 IF (J-1) 305,305,310
  305 IF (K-1) 306,306,307
  306 IP=3
      GO TO 324
  307 IF (K-KKK) 309,308,309
  308 IP = 13
      AZOH(K,J)=ZPINT
      GO TO 324
  309 IP=2
      GO TO 324
  310 IF (K-1) 311,311,314
  311 IF (J-JJJ) 313,312,312
  312 IP = 6
      GO TO 324
  313 IP = 5
      IPT1(K+1,J)=12
      GO TO 324
  314 IF (K-KKK) 321,315,315
  315 IF (J-JJJ) 317,316,316
  316 IF (JJJ-JJ2) 318,317,318
  317 IP=10
      AZOH(K,J)=ZPINT
      GO TO 324
  318 IP=8
      GO TO 324
  317 IF (J-JJ2) 319,318,320
  318 IP = 9
      AZOH(K,J)=ZPINT
      GO TO 324
```

```
  319 IP = 11
      AZDH(K,J)=ZDINT
      GO TO 324
  320 IP = 7
      GO TO 324
  321 IF (J-JJJ) 323,322,322
  322 IP = 10
      GO TO 324
  323 IP = 1
  324 IF (IPTA(K,J).EQ.0) IPTA(K,J)=IP
      RCUR = RCUR + DR(1)
  325 CONTINUE
      WRITE (NPR,7000) K,(IPTA(K,KK ), KK=1,JJJ)
      ZCUR = ZCUR + DZ(1)
  326 CONTINUE
C******
C  NOW COMPUTE CELL PSEUDO-MASS ZM
C******
      DO 400  J= 2,JJJ
      TEMP=AP(1,J)
  400 ZM(1,J) =ARHO(1)*(TEMP   -0.5*DR(1))
C *****
C
C   NOW GRID FOR BODY 2 IS CREATED
C
C *****
      ZCUR = ZCUR - DZ(1)
      KKK = KM(2)
      JJJ = JM(2)
      DO 349  K=1,KKK
      RCUR =0.0
      DO 348  J=1,JJJ
      BR  (K,J) = RCUR
      BZ  (K,J) = ZCUR
      BRH (K,J) = RCUR
      BZH (K,J) = ZCUR
      BRDH(K,J) = RDOT(2)
      BZDH(K,J) = ZDOT(2)
      IF (J-1) 330,330,328
  328 IF (K-1) 330,330,329
  329 BA(K,J)=AREA(2)
      BV(K,J)  = 1.0
      BP(K,J)=0.0
      BTRR(K,J)=0.0
      BTZZ(K,J)=0.0
      BTRZ(K,J)=0.0
      BTTT(K,J)=0.0
      BSRR(K,J)=0.0
      BSZZ(K,J)=0.0
      BSRZ(K,J)=0.0
      BSTT(K,J)=0.0
C *****
C  NOW THE POINT CONDITION CODE IS DETERMINED
C *****
  330 IF (J-1) 331,331,336
  331 IF (K-1) 332,332,333
  332 IP = 13
      BZDH(K,J)=ZDINT
      GO TO 347
  333 IF (K-KKK) 335,334,334
```

```
  334 IP = 4
      GO TO 347
  335 IP = 2
      GO TO 347
  336 IF (K-1) 337,337,340
  337 IF (J-JJJ) 339,338,338
  338 IF (JJJ-JM(1)) 840,839,840
  839 IP=10
      BZDH(K,J)=ZDINT
      GO TO 347
  840 IP=9
      BZDH(K,J)=ZDINT
      GO TO 347
  339 IP=11
      BZDH(K,J)=ZDINT
      GO TO 347
  340 IF (K-KKK) 344,341,341
  341 IF (J-JJJ) 343,342,342
  342 IP = 8
      GO TO 347
  343 IP = 7
      GO TO 347
  344 IF (J-JJJ) 346,345,345
  345 IP =10
      GO TO 347
  346 IP =1
  347 IPTB(K,J)=IP
      RCUR = RCUR +DR(2)
  348 CONTINUE
      WRITE (NPR,7000) K,(IPTB(K,KK),KK=1,JJJ)
      ZCUR = ZCUR +DZ(2)
  349 CONTINUE
C *****
C  NOW COMPUTE CELL PSEUDO-MASS ZM
C *****
      DO 401 J=2,JJJ
      TEMP=BR(1,J)
  401 ZM(2,J) = ARHO(2) * (TEMP   -0.5*DR(2))
C *****
C
C   INITIALISE CYCLE COUNTER, TIME, TIME STEPS,WAVE BOUNDARIES
C
C *****
      ICYCL =0
      TYME  =0
      DTN =DTH
      DTHW =DTH
      KMN =KINT -3
      KMX = 4
      JMX = JM(2) + 2
      IF (JMX.GT.JM(1)) JMX=JM(1)
 7000 FORMAT ('0',20I5)
      RETURN
      END
      SUBROUTINE LETGC
      IMPLICIT REAL*8 (A-H,O-Z,$), INTEGER (I-N)
      COMMON /CCM1/ AR  (24,46),   ARH (24,46),    ARDH(24,46),
     1              AZ  (24,46),   AZH (24,46),    AZDH(24,46),
     2              AA  (24,46),   AV  (24,46),    AP  (24,46),
     3              ATRR(24,46),   ATZZ(24,46),    ATRZ(24,46),
```

```
     4                ATTT(24,46),
     5                ASRR(24,46),  ASZZ(24,46),   ASRZ(24,46),
     6                ASTT(24,46)
      COMMON /CCM2/ BR   (46, 6),  BRH (46, 6),   BRDH(46, 6),
     1              BZ   (46, 6),  BZH (46, 6),   BZDH(46, 6),
     2              BA   (46, 6),  BV   (46, 6),  BP(46, 6),
     3              BTRR(46, 6),   BTZZ(46, 6),   BTRZ(46, 6),
     4              BTTT(46, 6),
     5              BSRR(46, 6),   BSZZ(46, 6),   BSRZ(46, 6),
     6              BSTT(46, 6)
      COMMON /CCM3/ TR   (24,46),  TRH (24,46),   TRDH(24,46),
     1              TZ   (24,46),  TZH (24,46),   TZDH(24,46),
     2   SR(46),  SZ(46),  TSR(46),  TSZ(46),  ZM(2,46),  SPTM(5),
     4 A1(2), A2(2), A3(2), A4(2), ALEN(2), AMAT(2), AMU(2), ANU(2),
     5 BK(2), CAPE(2), CDOT(2), DR(2), DZ(2), RAD(2), RHO(2), RDOT(2),
     6 RHOG(2), RHOT(2), TWMU(2), YT(2), ZDOT(2), ZV(2), VCON(2),
     7 CQ,DTN,DTH,DTHW,DTMIN,ZDINT,TYME,CHEKD,FACTR
      COMMON /ICCM1/  IPTA(24,46)
      COMMON /ICCM2/  IPTB(46, 6)
      COMMON /ICOM3/ ISHP(2), JM(2), KM(2),      KMID,
     1 IRUN,IDT1,IDT2,IDENT,IMAX,IPRNT,ICYCL,IMAP,NTT,KINT,KMN,KMX
      JJJ=JM(2)
      DO 350 J=1,JJJ
      AR(KMID,J)=BR(2,J)
      ARH (KMID,J)=BRH (2,J)
      ARDH(KMID,J)=BRDH(2,J)
      AZ  (KMID,J)=BZ  (2,J)
      AZH (KMID,J)=BZH (2,J)
      AZDH(KMID,J)=BZDH(2,J)
      ATRR(KMID,J)=BTRR(2,J)
      ATZZ(KMID,J)=BTZZ(2,J)
      ATRZ(KMID,J)=BTRZ(2,J)
      ATTT(KMID,J)=BTTT(2,J)
      ASRR(KMID,J)=BSRR(2,J)
      ASZZ(KMID,J)=BSZZ(2,J)
      ASRZ(KMID,J)=BSRZ(2,J)
      ASTT(KMID,J)=BSTT(2,J)
      AA  (KMID,J)=BA  (2,J)
      AV  (KMID,J)=BV  (2,J)
  350 AP  (KMID,J)=BP  (2,J)
C ****
C
C    FIND THE SWEEP BOUNDARIES
C
C ****
      IF (ZDOT(1)) 300,301,300
  300 KMN = 1
      JMX=JM(1)
      GO TO 303
  301 IF (KMN-1)    303,303,302
  302 KMN =KMN-1
  303 IF (JMX-JM(1)) 304,305,305
  304 JMX = JMX +1
  305 IF (KMX-KM(2)) 306,307,307
  306 KMX = KMX +1
C ****
C
C     CHECK IF THE TIME STEP HAS ATROPHIED BELOW THE ACCEPTABLE
C         LIMIT OF 10 PERCENT OF INITIAL TIME STEP
C
```

```
C ****
  307 IF (DTHW-DTMIN) 308,309,309
  308 IMAX = ICYCL + 1
      GO TO 310
C ****
C
C                    N            N+1/2
C     FIND NEXT DELT  AND DELT
C
C ****
  309 DTN = (DTF+DTHW)/2.0
      DTH = DTFW
      CHEKD=DTFW
C ****
C
C     INCREMENT THE CYCLE COUNTER AND THE TIME VALUE
C
C ****
  310 ICYCL = ICYCL +1
      TYME  = TYME  +DTH
      RETURN
      END
      SUBROUTINE MMOV(IB,DELZ,DELZH,     KLST,KEND,JM2)
      IMPLICIT REAL*8 (A-H,O-Z,$), INTEGER (I-N)
      COMMON /CCM1/ AR   (24,46),  ARH (24,46),   ARDH(24,46),
     1              AZ   (24,46),  AZH (24,46),   AZDH(24,46),
     2              AA   (24,46),  AV  (24,46),   AP  (24,46),
     3              ATRR(24,46),   ATZZ(24,46),   ATRZ(24,46),
     4              ATTT(24,46),
     5              ASRR(24,46),   ASZZ(24,46),   ASRZ(24,46),
     6              ASTT(24,46)
      COMMON /CCM2/ BR   (46, 6),  BRH (46, 6),   BRDH(46, 6),
     1              BZ   (46, 6),  BZH (46, 6),   BZDH(46, 6),
     2              BA   (46, 6),  BV  (46, 6),   BP(46, 6),
     3              BTRR(46, 6),   BTZZ(46, 6),   BTRZ(46, 6),
     4              BTTT(46, 6),
     5              BSRR(46, 6),   BSZZ(46, 6),   BSRZ(46, 6),
     6              BSTT(46, 6)
      COMMON /CCM3/ TR   (24,46),  TRH (24,46),   TRDH(24,46),
     1              TZ   (24,46),  TZH (24,46),   TZDH(24,46),
     2   SR(46),  SZ(46),  TSR(46),  TSZ(46),  ZM(2,46),  SPTM(5),
     4 A1(2), A2(2), A3(2), A4(2), ALFN(2), AMAT(2), AMU(2), ANU(2),
     5 BK(2), CAPE(2), CDCT(2), DR(2), DZ(2), RAD(2), RHO(2), RDOT(2),
     6 RHUG(2), RHLT(2), TWMU(2), YT(2), ZOUT(2), ZV(2), VCUN(2),
     7 CQ,DTN,DTF,DTHW,DTMIN,ZOINT,TYME,CHEKD,FACTR
      COMMON /ICCM1/  IPTA(24,46)
      COMMON /ICCM2/ IPTB(46, 6)
      COMMON /ICCM3/ ISHP(2), JM(2), KM(2),      KMID,
     1 IRUN,IDT1,IDT2,IDENT,IMAX,IPRNT,ICYCL,IMAP,NTT,KINT,KMN,KMX
      KSTRT=KLST+1
      DO 300 K=KSTRT,KEND
      DO 300 J=1,JM2
      BZ(K,J)=BZ(K,J) + DELZ
  300 BZH(K,J) = BZH(K,J) + DELZH
      RETURN
      END
      SUBROUTINE FRAC (KK,SFS,IPT,KUX,R,Z,RH,ZH,RDH,ZDH,TRR,TZZ,TRZ,TTT,
     1                 SRR,SZZ,SRZ,STT,P,JLST)
      IMPLICIT REAL*8 (A-H,O-Z,$),  INTEGER (I-N)
      DIMENSION                IPT(KUX,1),
```

```
     1   R(KUX,1), Z(KUX,1), RH(KUX,1), ZH(KUX,1), RDH(KUX,1),
     2 ZDH(KUX,1), TRR(KUX,1), TZZ(KUX,1), TRZ(KUX,1), TTT(KUX,1),
     3  SRR(KUX,1),  SZZ(KUX,1),  SRZ(KUX,1), STT(KUX,1), P(KUX,1)
      COMMON /CCM3/ TR  (24,46),  TRH (24,46),    TRDH(24,46),
     1              TZ  (24,46),  TZH (24,46),    TZDH(24,46),
     2   SR(46),  SZ(46),   TSR(46),   TSZ(46),   ZM(2,46),   SPTM(5),
     4 A1(2), A2(2), A3(2), A4(2), ALEN(2), AMAT(2), AMU(2), ANU(2),
     5 BK(2), CAPE(2), CDOT(2), DR(2), DZ(2), RAD(2), RHO(2), RDOT(2),
     6 RHUG(2), RHLT(2), TWMU(2), YT(2), ZOUT(2), ZV(2), VCON(2),
     7 CQ,DTN,DTF,DTHW,DTMIN,ZDINT,TYME,CHEKD,FACTR
      COMMON /ICCM3/ ISHP(2), JM(2), KM(2),      KMID,
     1 IRUN,IDT1,IDT2,IDENT,IMAX,IPRNT,ICYCL,IMAP,NTT,KINT,KMN,KMX
      NPR=6
      KCOL=KK
      IF (KCOL.EQ.1) RETURN
      DO 302 J=2,JLST
      SHR=TRZ(KK,J)
      IF (DABS(SHR)-SHS) 302,300,300
  300 IP = IPT(KK,J)
      JM1=J-1
      JP1=J+1
      IPL=IPT(KK,JM1)
      IF (((IP.GT.13).AND.(IP.LT.20)).OR.((IPL.GT.13).AND.(IPL.LT.20)))
     1  GO TO 302
      WRITE (NPR,100) KK,J
  100 FORMAT (1H0, '**** SHEAR FRACTURE **** AT K=',I5,5X,'J=',I5/)
      TR  (KCOL,J  ) =         R  (KK,J  )
      TZ  (KCOL,J  ) =         Z  (KK,J  )
      TRH (KCOL,J  ) =         RH (KK,J  )
      TZH (KCOL,J  ) =         ZH (KK,J  )
      TRDH(KCOL,J  ) =         RDH(KK,J  )
      TZDH(KCOL,J  ) =         ZDH(KK,J  )
      IF (IP.EQ.9) GO TO 9
      IF (IP.EQ.1) GO TO 1
      IF (IP.EQ.7) GO TO 7
      GO TO 302
    9 IPT(KK,J) = 14
      IPT(KK,JP1)=21
      GO TO 330
    1 IPT(KK,J)=15
      IPT(KK,JP1)=22
      GO TO 330
    7 IPT(KK,J)=16
      IPT(KK,JP1)=23
  330 CONTINUE
      DO 3000 II=1,2
      JJ=J+II-1
      KM1=KK-1
      KMIN1=KM1
      JM1=JJ-1
      TRRW=TRR(KK,JJ)
      TZZW=TZZ(KK,JJ)
      TRZW=TRZ(KK,JJ)
      TTTW=TTT(KK,JJ)
      IP=IPT(KK,JJ)
      R1=TR(KK,JJ)
      Z1=TZ(KK,JJ)
      IPLEF=IPT(KK-1,JJ)
      IF ((IPLEF.GT.13).AND.(IPLEF.LT.20)) GO TO 5101
      GO TO 5102
```

```
5101 Z2=TZ(KK-1,JJ)
     R2=TR(KK-1,JJ)
     GO TO 5103
5102 Z2=Z(KK-1,JJ)
     R2=R(KK-1,JJ)
5103 CONTINUE
     IPR=IPT(KM1,JM1)
     IF ((IPR.GT.13).AND.(IPR.LT.20)) GO TO 5019
     GO TO 5018
5019 Z3 = TZ(KMIN1,JM1)
     R3 = TR(KMIN1,JM1)
     GO TO 5020
5018 Z3=Z(KM1,JM1)
     R3=R(KM1,JM1)
5020 IPR=IPT(KK,JM1)
     IF ((IPR.GT.13).AND.(IPR.LT.20)) GO TO 5021
     GO TO 5022
5021 Z4=TZ(KK,JJ-1)
     R4=TR(KK,JJ-1)
     GO TO 5023
5022 Z4=Z(KK,JM1)
     R4=R(KK,JM1)
5023 CONTINUE
     CALL FREESL (1,R1,Z1,R2,Z2,R3,Z3,R4,Z4,TRRW,TZZW,TRZW,TTTW,SRRW,
    1SZZW,SRZW,STTW,KK,IP,PW)
     TRR(KK,JJ)=TRRW
     TZZ(KK,JJ)=TZZW
     TRZ(KK,JJ)=TRZW
     TTT(KK,JJ)=TTTW
     P(KK,JJ)=PW
     SRR(KK,JJ)=SRRW
     SZZ(KK,JJ)=SZZW
     SRZ(KK,JJ)=SRZW
     STT(KK,JJ)=STTW
3000 CONTINUE
 302 CONTINUE
     RETURN
     END
     SUBROUTINE FRACTN (TRR,TZZ,TRZ,TTT,K,KUX,R,Z,RH,ZH,RDH,ZDH,J1,
    1TSTR,SRR,SZZ,SRZ,STT,P,IPT)
     IMPLICIT REAL*8 (A-H,O-Z,$), INTEGER (I-N)
     DIMENSION                 IPT(KUX,1),
    1  R(KUX,1), Z(KUX,1), RH(KUX,1), ZH(KUX,1), RDH(KUX,1),
    2 ZDH(KUX,1), TRR(KUX,1), TZZ(KUX,1), TRZ(KUX,1), TTT(KUX,1),
    3  SRR(KUX,1),  SZZ(KUX,1),  SRZ(KUX,1), STT(KUX,1), P(KUX,1)
     COMMON /CCM3/ TR  (24,46),  TRH (24,46),   TRDH(24,46),
    1              TZ  (24,46),  TZH (24,46),   TZDH(24,46),
    2   SR(46),  SZ(46),   TSR(46),   TSZ(46),   ZM(2,46),   SPTM(5),
    4 A1(2), A2(2), A3(2), A4(2), ALFN(2), AMAT(2), AMU(2), ANU(2),
    5 BK(2), CAPF(2), CDOT(2), DR(2), DZ(2), RAD(2), RHO(2), ROOT(2),
    6 RHOG(2), RHCT(2), TWMU(2), YT(2), ZDOT(2), ZV(2), VCON(2),
    7 CQ,DTN,DTH,DTHA,DTMIN,ZDINT,TYME,CHEKD,FACTR
     NPR=6
     KP1=K+1
     DO 3000 J=2,J1
     STR=TRR(K,J)
     STZ=TZZ(K,J)
     STRZ=TRZ(K,J)
     FCON=0.5*(STR+STZ)
     TCON=0.5*(STZ-STR)
```

```
      IF (DABS(TCON)) 430,430,429
  429 SBC=STRZ/TCON
      PHI=DATAN(SBC)
      GO TO 431
  430 PHI=0.0
  431 ALPHA=DSIN(PHI)
      BETA=DCOS(PHI)
      TZZP=ECCN+TCON*BETA +STRZ*ALPHA
      TRRP=-TZZP+STZ+STR
      PHI=90.0*PHI/3.1415928
      IF (TZZP-TRRP) 302,303,303
  302 IF(TRRP) 312,312,314
  314 TNS=TRRP
      GO TO 322
  303 IF (TZZP) 312,312,324
  324 TNS=TZZP
      GO TO 322
  322 IF(TNS-TSTR) 312,326,326
  326 JM1=J-1
  328 WRITE (NPR,100) K,J,PHI
  100 FORMAT(1H0,'****SPALL****AT K=',I5,5X,'J='I5,'ANGLE='F8.2)
      IPT(K,J)=19
      TRR(K,J)=0.0
      TZZ(K,J)=0.0
      TRZ(K,J)=0.0
      PW=-TTT(K,J)/3.0
      P(K,J)=PW
      SRR(K,J)=PW
      STT(K,J)=-2.0*PW
      SRZ(K,J)=0.0
      SZZ(K,J)=PW
      TRR(KP1,J)=0.0
      TZZ(KP1,J)=0.0
      TRZ(KP1,J)=0.0
      PW=-TTT(K,J)/3.0
      P(KP1,J)=PW
      SRR(KP1,J)=PW
      SZZ(KP1,J)=PW
      SRZ(KP1,J)=0.0
      STT(KP1,J)=-2.0*PW

      ZO=Z(K,JP1)
      RO=R(KP1,J)
      ZA=Z(K-1,JP1)
      RA=R(K-1,JP1)
      ZB=Z(K-1,J)
      RB=R(K-1,J)

      ZC=Z(K,J)
      RC=R(K,J)
      IP=22
      TRRW=TRR(K,JP1)
      TZZW=TZZ(K,JP1)
      TRZW=TRZ(K,JP1)
      TTTW=TTT(K,JP1)
      CALL FREESU (1,RO,ZO,RA,ZA,RB,ZB,RC,ZC,TRRW,TZZW,TRZW,TTTW,
     1SRRW,SZZW,SRZW,STTW,K,IP,PW)
      TRR(K,JP1)=TRRW
      TZZ(K,JP1)=TZZW
      TRZ(K,JP1)=TRZW
```

```
      TTT(K,JP1)=TTTW
      P(K,JP1)=PW
      SRR(K,JP1)=SRRW
      SZZ(K,JP1)=SZZW
      SRZ(K,JP1)=SRZW
      STT(K,JP1)=STTW
      IPT(K,JM1)=18
      JP1=J+1
      IPT(K,JP1)=22
      IP1=IPT(KP1,J)
      IF (IP1.EQ.22) IPT(KP1,J)=20
      TR(K,J)=20
      TZ(K,J)=Z(K,J)
      TRH(K,J)=RH(K,J)
      TZH(K,J)=ZH(K,J)
      TRDH(K,J)=RDH(K,J)
      TZDH(K,J)=ZDH(K,J)
      TR(K,JM1)=R(K,JM1)
      TZ(K,JM1)=Z(K,JM1)
      TRH(K,JM1)=RH(K,JM1)
      TZH(K,JM1)=ZH(K,JM1)
      TRDH(K,JM1)=RDH(K,JM1)
      TZDH(K,JM1)=ZDH(K,JM1)
  312 CONTINUE
 3000 CONTINUE
      RETURN
      END
      SUBROUTINE GUTT (IB   ,IPT,R,Z,RH,ZH,RDH,ZDH,TRR,TZZ,TRZ,TTT,SRR,
     1                 SZZ,SRZ,STT,A,V,P,KUX,KMIN,KMAX,JMAX)
      IMPLICIT REAL*8 (A-H,O-Z,$), INTEGER (I-N)
      COMMON /CCM3/ TR  (24,46),  TRH (24,46),   TRDH(24,46),
     1              TZ  (24,46),  TZH (24,46),   TZDH(24,46),
     2   SR(46),  SZ(46),   TSR(46),   TSZ(46),   ZM(2,46),   SPTM(5),
     4 A1(2), A2(2), A3(2), A4(2), ALEN(2), AMAT(2), AMU(2), ANU(2),
     5 BK(2), CAPE(2), CDOT(2), DR(2), DZ(2), RAD(2), RHO(2), RDOT(2),
     6 RHOG(2), RHOT(2), TWMU(2), YT(2), ZDOT(2), ZV(2), VCON(2),
     7 CQ,DTN,DTH,DTHW,DTMIN,ZDINT,TYME,CHEKD,FACTR
      COMMON /ICCM3/ ISHP(2), JM(2), KM(2),      KMID,
     1 IRUN,IDT1,IDT2,IDENT,IMAX,IPRNT,ICYCL,IMAP,NTT,KINT,KMN,KMX
      DIMENSION
     1   IPT(KUX,1),
     1              R(KUX,1),       RH(KUX,1), RDH(KUX,1),
     2              Z(KUX,1),       ZH(KUX,1), ZDH(KUX,1),
     3  TRR(KUX,1),  TZZ(KUX,1),  TRZ(KUX,1),  TTT(KUX,1),
     2  SRR(KUX,1),  SZZ(KUX,1),  SRZ(KUX,1),  STT(KUX,1),
     5  A(KUX,1), V(KUX,1), P(KUX,1)
      NPR=6
      SHS =A1(IB)
C ****
C
C     INITIALISE TO ZERO VARIOUS VARIABLES
C
C ****
      TRRA =0.0
      TRRB =0.0
      TRRC=0.0
      TRRD=0.0
      TZZA=0.0
      TZZB=0.0
      TZZC=0.0
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      TZZD=0.0
      TRZA=0.0
      TRZB=0.0
      TRZC=0.0
      TRZD=0.0
      ACONA=0.0
      ACONB=0.0
      ACONC=0.0
      ACOND=0.0
      BCONA=0.0
      BCONB=0.0
      BCONC=0.0
      BCOND=0.0
      PCONA=0.0
      PCONB=0.0
      PCONC=0.0
      PCOND=0.0
      Z12=0.0
      Z23=0.0
      Z34=0.0
      Z41=0.0
      R12=0.0
      R23=0.0
      R34=0.0
      R41=0.0
      DO  3001 K= KMIN,KMAX
      KMIN1 = K-1
      KPLS1 = K+1
      KM1 = KMIN1
      KCUR= K
      KP1 = KPLS1
 4001 DO  3000 J=1,JMAX
      JM1 = J-1
      JP1 = J+1
      IP = IPT(K,J)
      IF (K.GT.1) IPLFF=IPT(KMIN1,J)
 5031 ITA=0
      ITB=0
      ITC=0
      ITD=0
      ITS=0
      ITDA=0
      ITDB=0
      ITDC=0
 5033 GO TO (5001,5001,5001,5001,5001,5001,5004,5004,5001,5001,5001,
     1       5001,5001,5001,5001,5004,5004,5001,5004,5001,5004,5001,
     2  5004,5004,5002,5004,5004,5002,5002),IP
 5001 R1 = R(KP1,J)
      Z1 = Z(KP1,J)
      GO TO 5004
 5002 IPR=IPT(KP1,J)
      IF ((IPR.GT.13).AND.(IPR.LT.20)) GO TO 5003
      GO TO 5001
 5003 R1 = TR(KPLS1,J)
      Z1 = TZ(KPLS1,J)
 5004 CONTINUE
      GO TO (5005,5005,5005,5005,5005,5006,5005,5006,5005,5006,5005,
     1       5005,5005,5006,5006,5006,5006,5005,5006,5005,5005,5005,
     2  5005,5005,5005,5005,5006,5226,5226),IP
 5005 R2 = R(KCUR,JP1)
```

```
      Z2 = Z(KCUR,JP1)
 5226 IPR=IPT(K,JP1)
      IF ((IPR.GT.13).AND.(IPR.LT.20)) GO TO 5227
      GO TO 5005
 5227 R2=TR(K,JP1)
      Z2=TZ(K,JP1)
 5006 CONTINUE
      GO TO (5007,5007,5010,5007,5010,5010,5007,5007,5007,5007,5007,
     1   5007,5007,5007,5007,5007,5010,5007,5007,5009,5007,5007,5007,
     2   5008,5008,5008,5010,5010,5008),IP
 5007 R3=R(KM1,J)
      Z3=Z(KM1,J)
      GO TO (5036,5034),IB
 5036 IF ((K.EQ.KMIN).AND.(K.GT.1)) GO TO 5035
 5034 CONTINUE
      GO TO 5010
 5035 SR(J)=R3
      SZ(J)=Z3
      GO TO 5010
 5009 R3=TR(KM1,J)
      Z3=TZ(KM1,J)
      GO TO 5010
 5008 IPR=IPT(KM1,J)
      IF ((IPR.GT.13).AND.(IPR.LT.20)) GO TO 5009
 6010 R3=RTEMP
      Z3=ZTEMP
 5010 CONTINUE
      GO TO (5011,5014,5014,5014,5011,5011,5011,5011,5011,5011,5011,
     1         5011,5014,5011,5011,5011,5014,5011,5012,5013,5013,5013,
     2   5013,5014,5014,5014,5014,5014,5013),IP
 5011 IPR=IPT(K,JM1)
      IF ((IPR.GT.13).AND.(IPR.LT.20)) GO TO 5013
 5012 R4 = R(KCUR,JM1)
      Z4 = Z(KCUR,JM1)
      GO TO 5014
 5013 R4 = TR(K,JM1)
      Z4 = TZ(K,JM1)
 5014 CONTINUE
      GO TO (4000,4003),IB
 4000 IF (K-1) 4003,4003,4001
 4001 IF (K-KMIN) 4003,4002,4003
 4002 SR(J)=R(KM1,J)
      SZ(J)=Z(KM1,J)
 4003 CONTINUE
  305 GO TO (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,
     1  23,24,25,26,27,28,29),IP
    1 ITA=1
      ITB=1
      ITC=1
      ITD=1
      GO TO 704
    2 ITS=1
      ITA=1
      ITB=1
      GO TO 702
    3 ITS=1
      ITA=1
      GO TO 701
    4 ITS=1
      ITB=1
```

```
      ITOA=1
      GO TO 702
    5 ITA=1
      ITD=1
      ITOC=1
      GO TO 704
    6 ITD=1
      ITOC=2
      GO TO 704
    7 ITB=1
      ITC=1
      ITOA=1
      GO TO 703
    8 ITC=1
      ITOB=1
      GO TO 703
    9 ITB=1
      ITC=1
      ITD=1
      ITOA=1
      GO TO 704
   10 ITC=1
      ITD=1
      ITOB=1
      GO TO 704
   11 CONTINUE
      GO TO 1
   12 CONTINUE
      GO TO 1
   13 ITA=1
      ITB=1
      ITS=1
      GO TO 702
   14 CONTINUE
      GO TO 10
   15 CONTINUE
      GO TO 10
   16 CONTINUE
      GO TO 8
   17 CONTINUE
      GO TO 3000
   18 CONTINUE
      GO TO 9
   19 CONTINUE
      GO TO 8
   20 CONTINUE
      GO TO 1
   21 CONTINUE
      GO TO 7
   22 CONTINUE
      GO TO 1
   23 CONTINUE
      GO TO 7
   24 CONTINUE
      ITB=1
      ITOA=1
      GO TO 702
   25 ITA=1
      ITB=1
      GO TO 702
```

```
   26 CONTINUE
      GO TO 24
   27 CONTINUE
      GO TO 3000
   28 ITA=1
      GO TO 701
   29 ITA=1
      ITB=1
      ITD=1
      ITOC=3
  704 Z41 =Z4-Z1
      R41 =R4-R1
      TRRD = TRRA
      TZZD = TZZA
      TRZD = TRZA
      PCOND= PCCNA
      ACOND= ACONA
      BCOND= BCCNA
      IF (ITCC.EQ.3) GO TO 702
      IF (ITOC-2) 306,313,306
  306 IF (ITCC-1) 703,701,703
  703 Z34=Z3-Z4
      R34=R3-R4
      TRRC = TRRB
      TZZC = TZZB
      TRZC = TRZB
      PCONC= PCCNB
      ACONC= ACCNB
      BCONC= BCCNB
      IF (ITCB-1) 307,313,307
  307 IF (ITOB-2) 702,701,702
  702 Z23 = Z2-Z3
      R23 = R2-R3
      TRRB = TRR(KCUR,JP1)
      TZZB = TZZ(KCUR,JP1)
      TRZB = TRZ(KCUR,JP1)
      AN   =  A   (KCUR,JP1)
      AMASS =  AN/ZM(IB,JP1)
      PCONB =  RHO(IB)*AN/ V(KCUR,JP1)
      ACONB =  TRZB*AMASS
      BCONB = (TRRB - TTT(KCUR,JP1))*AMASS
  308 IF (ITCA-1)  701,313,701
  701 Z12= Z1-Z2
      R12= R1-R2
      TRRA= TRR(KP1,JP1)
      TZZA= TZZ(KP1,JP1)
      TRZA= TRZ(KP1,JP1)
      AN  = A  (KP1,JP1)
      GO TO (309,311),IB
  309 IF (K-KINT) 311,310,311
  310 AMASS = AN/ZM(2,JP1)
      PCONA = RHO(2)*AN/ V(KP1,JP1)
      GO TO 312
  311 AMASS = AN/ZM(IB,JP1)
      PCONA = RHO(IB)*AN/V(KP1,JP1)
  312 ACONA = TRZA*AMASS
      BCONA =(TRRA-TTT(KP1,JP1))*AMASS
  313 IF (IP.GT.23) GO TO 5016
C ****
C
```

```
C     EQUATIONS OF MOTION
C
C ****
 5015 ZN=Z(KCUR,J)
      RN= R(KCUR,J)
      RDHF=RDH(KCUR,J)
      ZDHF=ZDH(KCUR,J)
      GO TO 5017
 5016 ZN=TZ(K,J)
      RN=TR(K,J)
      RDHF=TRDH(K,J)
      ZDHF=TZDH(K,J)
 5017 CONTINUE
      PHI= (ITA*PCONA + ITB*PCONB + ITC*PCONC + ITD*PCOND)*1.0012/4.0
      RZDCN = DTN/(2*PHI)
      IF (ITS-1) 315,314,315
  314 RNW=0.0
      RNH=0.0
      RDNH=0.0
      ALFA=0.0
      GO TO 318
  315 TCON= 1.0/(ITA+ITB+ITC+ITD)
      ALFA= TCON*(ITA*ACONA + ITB*ACONB + ITC*ACONC + ITD*ACOND)
      BETA= TCON*(ITA*BCONA + ITB*BCONB + ITC*BCONC + ITD*BCOND)
      RDNH= RDHF       + RZDCN*(ITA*(TRRA*Z12-TRZA*R12) + ITB*(TRRB*Z23
     1        -TRZB*R23) + ITC*(TRRC*Z34-TRZC*R34) + ITD*(TRRD*Z41-TRZD*
     2         R41)) +   DTN*BETA
      RDEF = RDNH*DTH
      RNW = RN+RDEF
      IF (DABS(RDEF)-1.0D-07) 316,316,317
  316 RNW = RN
  317 RNH = (RNW+RN)/2.0
  318 ZDNH= ZDHF       -RZDCN*(ITA*(TZZA*R12-TRZA*Z12) + ITB*(TZZB*R23
     1        -TRZB*Z23) + ITC*(TZZC*R34-TRZC*Z34) + ITD*(TZZD*R41-TRZD*
     2         Z41)) +   DTN*ALFA
      ZDEF =ZDNH*DTH
      ZNW = ZN+ZDEF
      IF (DABS(ZDEF)-1.0D-07) 319,319,320
  319 ZNW = ZN
  320 ZNH = (ZN +ZNW)/2.0
      IF (ITC)  322,321,322
  321 PHI=0.0
      GO TO 839
C ****
C
C     NOW THE STRESSES ARE COMPUTED
C
C ****
  322 Z1= ZNW
      R1= RNW
      IF ((IPLEF.GT.13).AND.(IPLEF.LT.20)) GO TO 5101
      GO TO 5102
 5101 Z2=TSZ(J)
      R2=TSR(J)
      RH2=TRH(KM1,J)
      ZH2=TZH(KM1,J)
      RDH2=TRDH(KM1,J)
      ZDH2=TZDH(KM1,J)
      GO TO 5103
 5102 Z2=SZ(J)
```

NOT REPRODUCIBLE

```
      R2=SR(J)
      RH2=RH(KM1,J)
      ZH2=ZH(KM1,J)
      RDH2=RDH(KM1,J)
      ZDH2=ZDH(KM1,J)
 5103 CONTINUE
      IPR=IPT(KM1,JM1)
      IF ((IPR.GT.13).AND.(IPR.LT.20)) GO TO 5019
      GO TO 5018
 5019 Z3 = TZ(KMIN1,JM1)
      R3 = TR(KMIN1,JM1)
      ZH13 = ZNH-TZH(KMIN1,JM1)
      RH13 = RNH-TRH(KMIN1,JM1)
      ZDH13 = ZDNH -TZDH(KMIN1,JM1)
      RDH13 = RDNH -TRDH(KMIN1,JM1)
      GO TO 5020
 5018 Z3=Z(KM1,JM1)
      R3=R(KM1,JM1)
      ZH13 =ZNH - ZH(KM1,JM1)
      RH13 =RNH - RH(KM1,JM1)
      ZDH13=ZDNH -ZDH(KM1,JM1)
      RDH13=RDNH -RDH(KM1,JM1)
 5020 IPR=IPT(K,JM1)
      IF ((IPR.GT.13).AND.(IPR.LT.20)) GO TO 5021
      GO TO 5022
 5021 Z4=TSZ(JM1)
      R4=TSR(JM1)
      ZH42 = TZH(K,JM1) -ZH2
      RH42 = TRH(K,JM1) -RH2
      RDH42 = TRDH(K,JM1) - RDH2
      ZDH42 = TZDH(K,JM1) - ZDH2
      GO TO 5023
 5022 Z4 = SZ(JM1)
      R4 = SR(JM1)
      RH42 =RH(KCUR,JM1)-RH2
      ZH42 =ZH(KCUR,JM1)-ZH2
      ZDH42=ZDH(KCUR,JM1) - ZDH2
      RDH42=RDH(KCUR,JM1) - RDH2
 5023 CONTINUE
C ****
C
C     COMPUTE THE AREA AND RELATIVE VOLUME AT NEW TIME
C
C ****
      AN= A(KCUR,J)
      VN= V(KCUR,J)
      Z42=Z4-Z2
      R42=R4-R2
      A124=0.5*(Z4*(R1-R2)-Z1*R42+Z2*(R4-R1))
      A234=0.5*(Z4*(R2-R3)+Z2*(R3-R4)+Z3*R42)
      AW=A124+A234
      AH=0.5*(AW+AN)
      VW=RHOT(IB) *(A124*(R4+R1+R2)+A234*(R3+R4+R2))/ZM(IB,J)
      VH=0.5*(VW+VN)
      DELVV=(VW-VN)/VH
      IF (DABS(DELVV)-1.0D-07) 323,323,324
  323 DELVV=0.0
C ****
C
C     COMPUTE THE STRAIN INCREMENTS
```

```
C
C ****
  324 ECON =0.5*DTH/AH
      EZZH = ECCN * (ZDH42*RH13 - RH42*ZDH13)
      ERRH =-ECCN * (RDH42*ZH13 - ZH42*RDH13)
      ERZH = ECCN *(RDH42*RH13-RH42*RDH13-ZDH42*ZH13+ZDH13*ZH42)
      ETTH = DELVV-EZZH-ERRH
      SCONC = DELVV/3.0
      SZZN  = SZZ(KCUR,J)
      SRRN  = SRR(KCUR,J)
      SRZN  = SRZ(KCUR,J)
      ALFA  = 0.5*ECON*(RDH42*RH13-RH42*RDH13+ZDH42*ZH13-ZH42*ZDH13)
      BETA  = 2.0*SRZN*ALFA
C ****
C
C     COMPUTE NEW STRESS DEVIATORS
C
C ****
      SZZW  = SZZN + TWMU(IB)*(EZZH-SCONC)+BETA
      SRRW  = SRRN + TWMU(IB)*(ERRH-SCONC)-BETA
      SRZW  = SRZN + AMU(IB) *ERZH + (SRRN-SZZN)*ALFA
      STTW  = STT(KCUR,J)+ TWMU(IB)*(ETTH-SCONC)
C ****
C
C     COMPUTE THE ARTIFICIAL VISCOSITY TERMS
C
C ****
      IF (DELVV) 326,325,325
  325 QQNH = 0.0
      GO TO 327
  326 VDOV = DELVV/DTH
      QQNH =  (AH*RHO(IB)/VH)*(CQ*VDOV)**2
C ****
C
C     USE THE EQUATION OF STATE TO COMPUTE  THE PRESSURE INCREMENT
C
C ****
  327 DELP = -BK(IB)*DELVV
      PW   =  P(KCUR,J)+DELP
C ****
C     TOTAL STRESSES ARE COMPUTED NEXT
C ****
      TZZW = SZZW-PW-QQNH
      TRRW = SRRW-PW-QQNH
      TRZW = SRZW
      TTTW = STTW-PW-QQNH
C ****
C
C     CALL THE FREE SURFACE ROUTINE TO ADJUST THE STRESS VALUES
C
C ****
      CALL FREESU(IB,R1,Z1,R2,Z2,R3,Z3,R4,Z4,TRRW,TZZW,TRZW,TTTW,
     1  SRRW,SZZW,SRZW,STTW,K,IP,PW)
C ****
C
C     COMPUTE THE PRINCIPAL STRESSES
C
C ****
  328 ECON = 0.5*(TZZW+TRRW)
      TCCN = 0.5*(TZZW-TRRW)
```

```
      IF (DABS(TCCN)) 330,330,329
  329 PHI=DATAN(TRZW/TCCN)
      GO TO 331
  330 PHI=0.0
  331 ALFA =DSIN(PHI)
      BETA =DCOS(PHI)
      TZZP = ECCN + TCON*BETA +TRZW*ALFA
      TRRP = -TZZP +TZZW + TRRW
      PHI  =90.0*PHI/3.1415928
C ****
C
C     CHECK FOR TIME STEP STABILITY
C
C ****
  332 RR13=  (R1-R3)**2 +(Z1-Z3)**2
      RR24=   R42*R42 + Z42*Z42
      IF (RR13 - RR24) 333,333,334
  333 DELR =DSQRT(RR13)
      GO TO 335
  334 DELR =DSQRT(RR24)
  335 CHEKD=(DELR/CDOT(IB))*FACTR
  839 CONTINUE
      GO TO (336,339),IB
  336 IF (K-KMIN)337,337,339
  337 IF (J-1)338,339,339
  338 KT=K
      JT=1
      DTHW=CHEKD
  339 IF (CHEKD-DTHW) 340,340,341
  340 KT=K
      JT=J
      IBT=IB
      DTHW = CHEKD
  341 IF (MOD(ICYCL,IPRNT))  351,342,351
  342 IF (J-1)  350,343,350
  343 IF  (IB-1) 346,344,346
  344 IF  (K-KMIN) 345,345,349
C ****
C
C     FIND THE WAVE FRONT POSITION
C
C ****
  345 WF = TYME*CDOT(1)
      KWF =IDINT(WF/DZ(1)+0.5)
      WRITE (NPR,100) ICYCL,TYME,WF,KWF,DTH,DTHW,IBT,KT,JT
  346 IF (IB-2) 349,347,349
  347 IF (K-2)  349,348,349
  348 WRITE (NPR,101) KINT
  349 WRITE (NPR,102) IB,K
      TZZP = 0.0
      TRRP = 0.0
      PHI = 0.0
      TZZW = 0.0
      TRRW = 0.0
      TRZW = 0.0
      TTTW = 0.0
C ****
C
C      WRITE OUT THE RESULTS
C
```

```
C ****
  350 WRITE (NPR,103) J,ZNW,RNW,TZZW,TRRW,TRZW,TTTW,TZZP,TRRP,PHI
C ****
C
C     TRANSFER DATA (GEOMETRIC) OF K-1 POINTS FROM TEMPORARY STORAGE
C        TO PERMANENT STORAGE
C
C ****
  351 IF (K-1) 353,353,352
  352 IF (IP.GT.23) GO TO 6001
      IF ((IP.GT.13).AND.(IP.LT.20)) GO TO 6002
      R(KM1,J)=SR(J)
      Z(KM1,J)=SZ(J)
      SR(J)=RNW
      SZ(J)=ZNW
      IF ((IPLEF.GT.13).AND.(IPLEF.LT.20)) GO TO 6003
      GO TO 6009
 6001 IF ((IPLEF.GT.13).AND.(IPLEF.LT.20)) GO TO 6004
      GO TO 6005
 6004 TR(KM1,J)=TSR(J)
      TZ(KM1,J)=TSZ(J)
 6005 TSR(J)=RNW
      TSZ(J)=ZNW
      GO TO 6009
 6002 IF ((IPLEF.GT.13).AND.(IPLEF.LT.20)) GO TO 6007
 6006 RTEMP=R(KM1,J)
      ZTEMP=Z(KM1,J)
 6007 R(KM1,J)=SR(J)
      Z(KM1,J)=SZ(J)
  353 SR(J)=RNW
      SZ(J)=ZNW
      GO TO 6009
 6003 TR(KM1,J)=TSR(J)
      TZ(KM1,J)=TSZ(J)
 6009 CONTINUE
      IF (IP.GT.23) GO TO 5026
      RH(KCUR,J)=RNH
      ZH(KCUR,J)=ZNH
      RDH(KCUR,J)=RDNH
      ZDH(KCUR,J)=ZDNH
      GO TO 5027
 5026 TRH(K,J)=RNH
      TZH(K,J)=ZNH
      TRDH(K,J) = RDNH
      TZDH(K,J) = ZDNH
 5027 CONTINUE
      IF (ITC-1)3000,354,3000
  354 A(KCUR,J)=AW
      V(KCUR,J)=VW
      P(KCUR,J)=PW
      SZZ(KCUR,J)=SZZW
      SRR(KCUR,J)=SRRW
      SRZ(KCUR,J)=SRZW
      STT(KCUR,J)=STTW
      TZZ(KCUR,J)=TZZW
      TRR(KCUR,J)=TRRW
      TRZ(KCUR,J)=TRZW
      TTT(KCUR,J)=TTTW
      IF (IP.GT.23) GO TO 300
      IF ((IP.GT.13).AND.(IP.LT.20)) GO TO 5030
```

```
      GO TO 3000
 5030 IP=IP+10
      GO TO 5031
 3000 CONTINUE
      IF (IB.EQ.1)
     1CALL FRAC(KM1,SHS,IPT,KUX,R,Z,RH,ZH,RDH,ZDH,TRR,TZZ,TRZ,TTT,
     2 SRR,SZZ,SRZ,STT,P, JM(1))
      IF (IB.EQ.1) CALL FRACTN (TRR,TZZ,TRZ,TTT,K,KUX,R,Z,RH,ZH,RDH,ZDH,
     1JM(1), YT(1),SRR,SZZ,SRZ,STT,P,IPT)
 3001 CONTINUE
      K=KMAX
C     THIS IS THE END OF THE K-LOOP
C *****
C
C  NOW K IS EQUAL TO KMAX
C
C *****
      FCON2=ZNW-ZN
      TCON2=ZNH-ZHM
      DO 374 J=1,JMAX
      Z(KCUR,J) =SZ(J)
      R(KCUR,J) =SR(J)
      GO TO (5032,374),IB
 5032 IF (IPT(KMAX,J)-13) 374,374,5033
 5033 TZ(KMAX,J) = TSZ(J)
      TR(KMAX,J) = TSR(J)
  374 CONTINUE
      IF (IB.EQ.1)
     1CALL FRAC(KMAX,SHS,IPT,KUX,R,Z,RH,ZH,RDH,ZDH,TRR,TZZ,TRZ,TTT,
     2  SRR,SZZ,SRZ,STT,P,JM(1))
      GO TO (380,375),IB
  375 IF (K-KM(2)) 377,380,380
  377 KKK=KM(2)
      JJJ=JM(2)
      CALL MMCV( 2,FCON2,TCON2,       KMAX,KKK,JJJ)
  380 CONTINUE
  100 FORMAT (1H1,5HCYCLE,I4,3X,6HTIME=,F8.4,3H MS,3X,9HTRC(1) =,F8.4,
     1  4H IN ,3X,7HK(TC)=,I6//13X,6HDELT =,F8.4,3H MS,3X,
     2  8HDELTMN =,F8.4,3H MS,4X,7HR,K,J =,I2,2I4)
  101 FORMAT (16H INTERFACE =COL.,I4)
  102 FORMAT (7H REGION,I2,3X,6HK COL.,I5//3X,1HJ,11X,1HZ,11X,1HR,5X,
     1         7HSIG(ZZ),5X,7HSIG(RR),5X,7HSIG(RZ),5X,7HSIG(TT),6X,
     2         6HSIG(1),6X,6HSIG(2),9X,3HPHI/4H ROW,2(8X,4H[. .]),
     3         6(9X,3HPS[),9X,3HPHI)
  103 FORMAT (I4,2F12.8,7F12.0)
      RETURN
      END
      SUBROUTINE FREESU (IB,RO,ZO,RA,ZA,RB,ZB,RC,ZC,TRR .      . . .  TR
     1,SRRW,SZZW,SRZW,STTW,K,IP,PW)
      IMPLICIT REAL*8 (A-H,O-Z,$), INTEGER (I-N)
      NPR=6
      PIE=3.1415928
      GO TO (1,1,1,1,1,1,7,8,1,10,1,12,1,13,10,8,1,1. . . . .  . .
     11,1,1,1,1,1),IP
    8 TRRW=0.0
      TZZW=0.0
      TRZW=0.0
      PW=-TTTW/3.0
      STTW=-2.0*PW
      SRZW=0.0
```

```
      SZZW=PW
      GO TO 1
    7 Z1=ZO
      R1=RO
      Z2=ZC
      R2=RC
      GO TO 300
   10 IF ((K.EQ.2).AND.(IB.EQ.1)) GO TO 9
      Z1=ZO
      R1=RO
      R2=RA
      Z2=ZA
      R2=RA
      GO TO 300
   12 Z1=ZA
      R1=RA
      Z2=ZB
      R2=RB
      GO TO 300
   22 Z1=ZB
      R1=RB
      Z2=ZC
      R2=RC
      GO TO 300
  300 D1=R1-R2
      IF (DABS(D1)-1.0D-05) 301,301,302
  301 ETA=0.0
      GO TO 310
  302 D2=Z1-Z2
      IF (DABS(D2)-1.0D-05) 303,303,304

  303 ETA=PIE/2.0
      GO TO 310
  304 ETA=DATAN((R1-R2)/(Z1-Z2))
  310 ALPHA=2.0*ETA
      TSTW=((TRRW+TZZW)/2.0)+0.5*(TZZW-TRRW)*DCOS(ALPHA)+TRZW*
     1DSIN(ALPHA)
      TRRW=0.5*TSTW*(1.0-DCOS(ALPHA))
      TZZW=0.5*TSTW*(1.0+DCOS(ALPHA))
      TRZW=0.5*TSTW*DSIN(ALPHA)
      PW=-(TRRW+TZZW+TTTW)/3.0
      SRRW=TRRW+PW
      SZZW=TZZW+PW
      SRZW=TRZW
      STTW=TTTW+PW
    1 CONTINUE
      RETURN
      END
//GO.SYSIN  DD *
      1
      6001       0711         70     0.5
      2          1            31          1
      1000
      ICE        0.29 273000.0        0.92      140.0
  72.0
      2          0.0          0.0        45          22      0.125      0.125
      ALMN       0.29 1000000.0        2.7  150000.0
 75000.0
      1          0.0         -7.7         5          45      0.125      0.125
/*
```

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